Effective Field Theories for Charmonium

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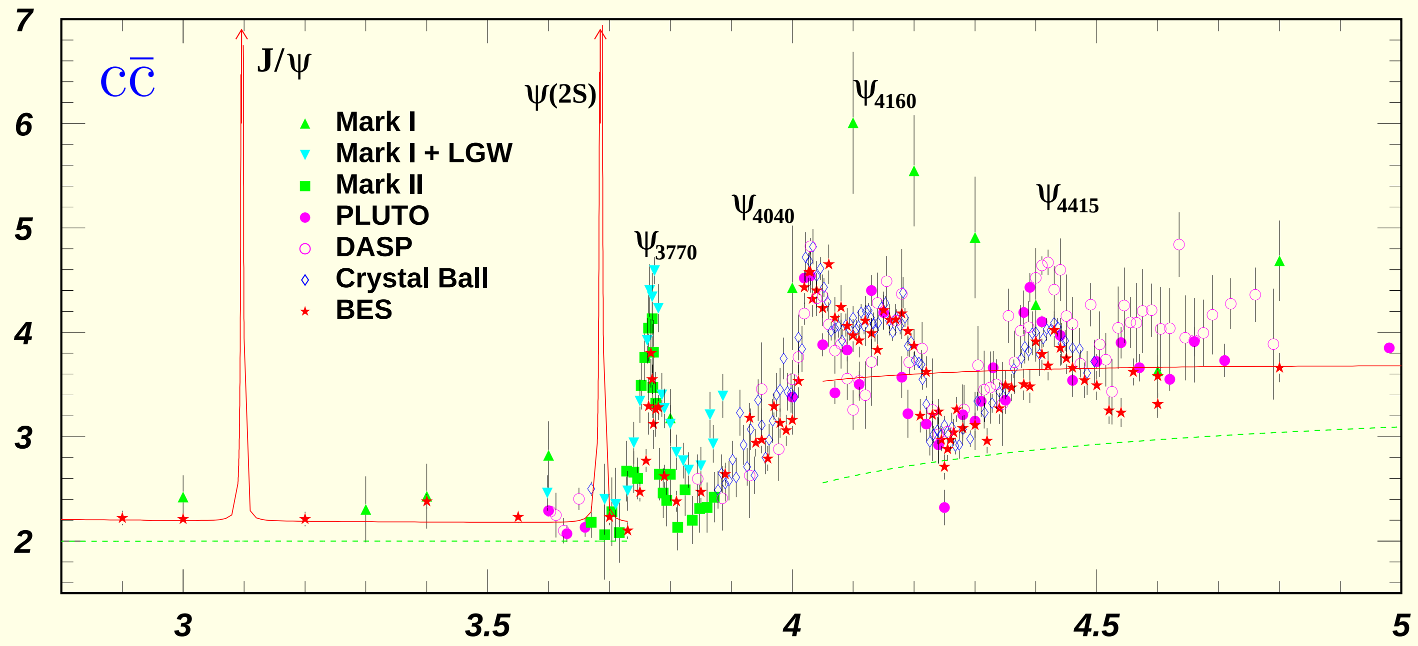
Quarkonium Working Group

Motivation



- The system is characterized by two expansion parameters: α_s and v.
  (i) hierarchy of scales ($\Rightarrow$ factorizaton/effective field theories)
  (ii) some of the scales are perturbative.
- For these same reasons, charmonium (and quarkonium) are systems where low energy QCD may be studied in a systematic way (e.g. non-perturbative matrix elements, QCD vacuum, confinement, exotica, ...)

Summary

1. Effective Field Theories: NRQCD, pNRQCD

2. Spectroscopy

2.1 Charm mass

2.2 Higher resonances: pNRQCD potentials

2.3 New Spectroscopy

3. Annihilations

3.1 Inclusive decays

3.2 Electromagnetic decays

4. Production

4.1 Polarization

5. Prospects of $p\bar{p}$ at Fermilab

6. Conclusion

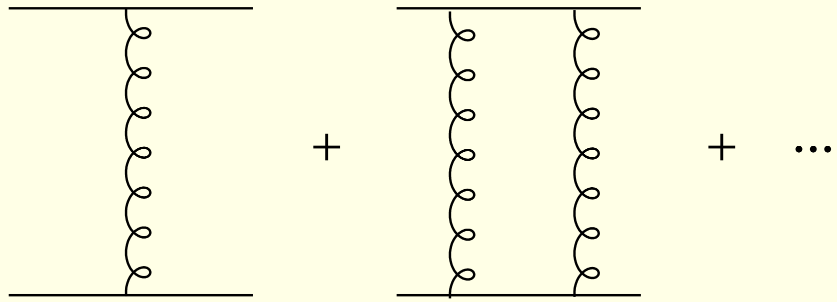
1. EFTs

Quarkonium Scales

Apart from α_s, another small parameter shows up near threshold:

$$E \approx 2m + \frac{p^2}{m} + \ldots \qquad \text{with} \quad v = \frac{p}{m} \ll 1$$

- The perturbative expansion breaks down when $\alpha_s \sim v$:

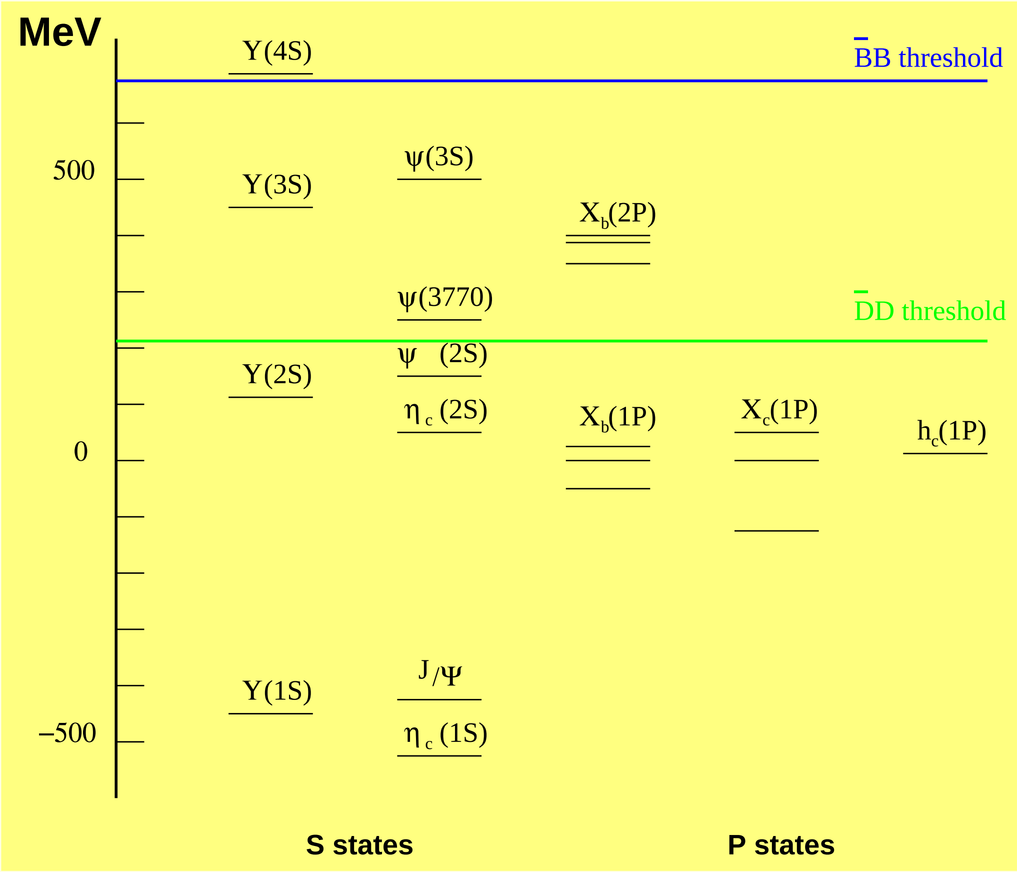


$$\approx \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

$$\alpha_s \left(1 + \frac{\alpha_s}{v} + \ldots\right)$$

- The system is non-relativistic : $p \sim mv$ and $E = \dfrac{p^2}{m} + V \sim mv^2$.

Quarkonium Scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

The mass scale is perturbative:

$m_b \simeq 5$ *GeV*, $m_c \simeq 1.5$ *GeV*

The system is non-relativistic:

$\Delta_n E \sim mv^2$, $\Delta_{\rm fs} E \sim mv^4$

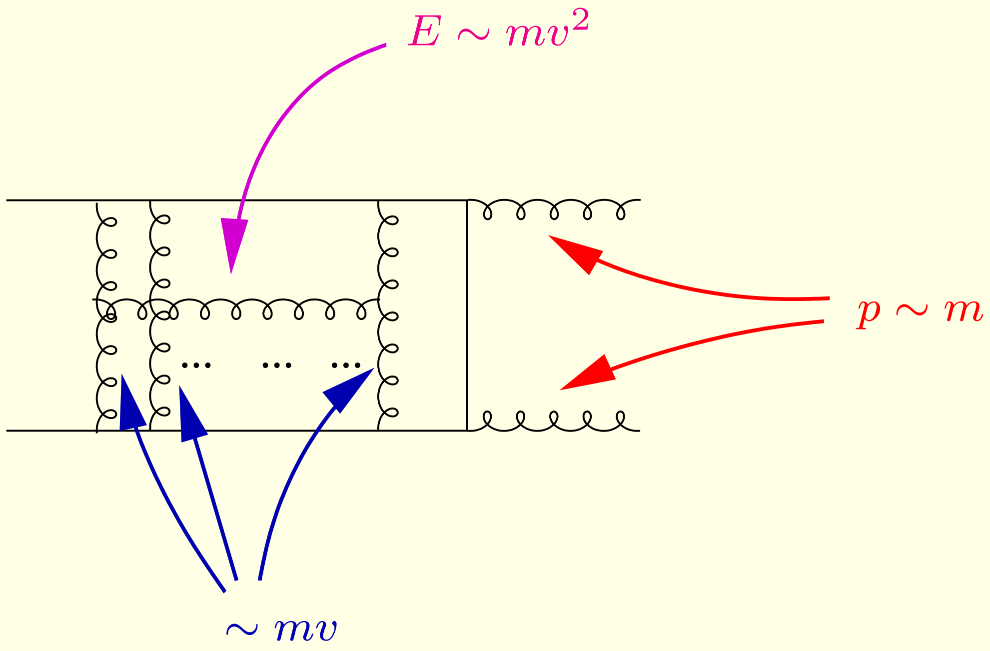
$v_b^2 \simeq 0.1$, $v_c^2 \simeq 0.3$

Non-relativistic bound states are characterized by at least three energy scales

$m \gg mv \gg mv^2 \quad v \ll 1$

Quarkonium Scales

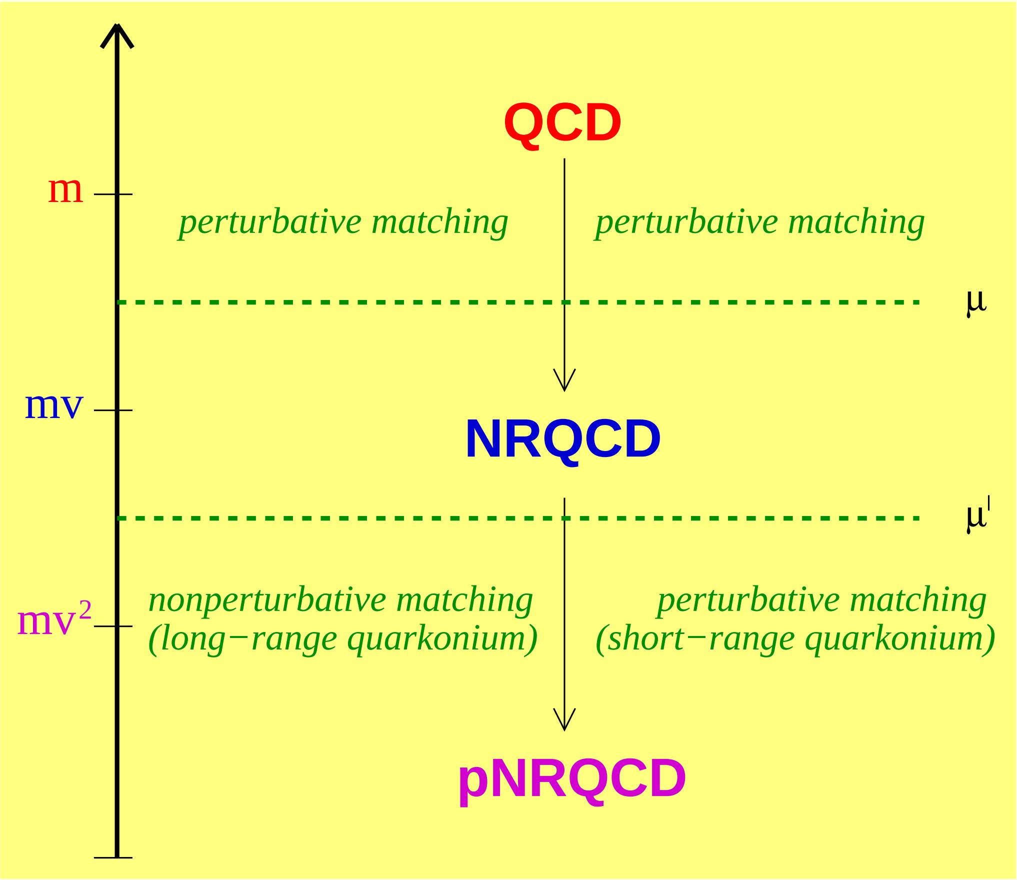
Scales get entangled.



Effective Field Theories for Quarkonium

Whenever a system H, described by $\mathcal{L}_{\rm QCD}$, is characterized by 2 scales $\Lambda \gg \lambda$, observables may be calculated by expanding one scale with respect to the other.

An effective field theory makes the expansion in λ/Λ explicit at the Lagrangian level.



$$\frac{\lambda}{\Lambda} = \frac{mv}{m}$$

$$\frac{\lambda}{\Lambda} = \frac{mv^2}{mv}$$

Charmonium Scales

$$m_c \approx 1.5 \text{ GeV} \gg \Lambda_{\mathrm{QCD}}$$

$$m_c v \approx 0.8 \text{ GeV} > \Lambda_{\mathrm{QCD}} \quad \text{for } J/\psi, \eta_c$$

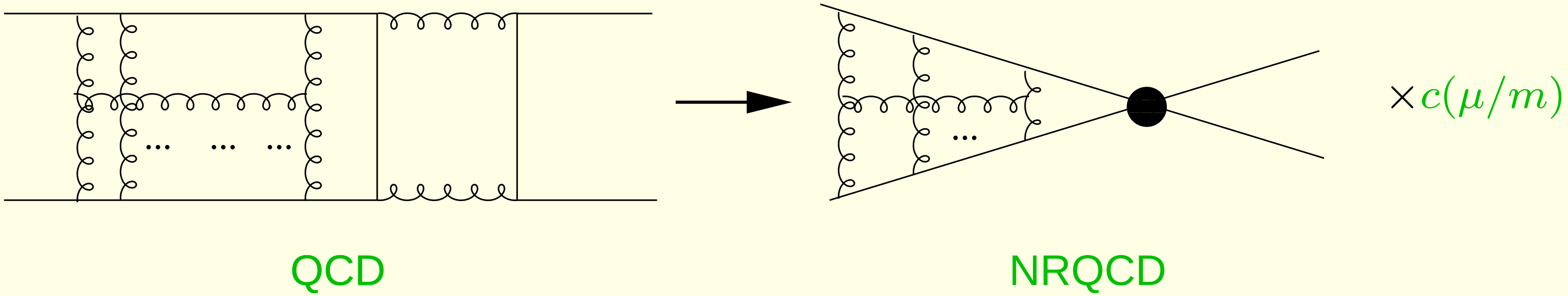
$$m_c v \sim \Lambda_{\mathrm{QCD}} \quad \text{for all higher resonances}$$

As a consequence:

- annihilation, production, hard scale processes happen at a perturbative scale;
- the bound state is perturbative (i.e. Coulombic) perhaps only for the J/ψ, η_c;
- for all other charmonium resonances the bound state is non-perturbative. It will be described by matrix elements, (confining) potentials to be determined on the lattice.

NRQCD

NRQCD is the EFT that follows from QCD when $\Lambda = m$



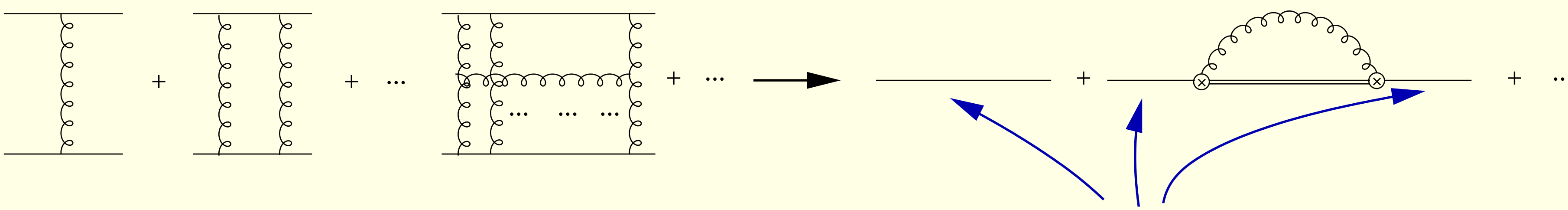
- The matching is perturbative.
- The Lagrangian is organized as an expansion in $1/m$ and $\alpha_s(m)$:

$$\mathcal{L}_{\rm NRQCD} = \sum_n c(\alpha_s(m/\mu)) \times O_n(\mu, \lambda)/m^n$$

Suitable to describe annihilation and production of quarkonium.

pNRQCD

pNRQCD is the EFT for heavy quarkonium that follows from NRQCD when $\Lambda = \frac{1}{r} \sim mv$



$$\frac{1}{E - p^2/m - V(r)}$$

NRQCD pNRQCD

- The Lagrangian is organized as an expansion in $1/m$, r, and $\alpha_s(m)$:

$$\mathcal{L}_{\rm pNRQCD} = \sum_k \sum_n \frac{1}{m^k} \times c_k(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

1. Spectroscopy

Low lying $c\bar{c}$

Low lying $c\bar{c}$ states are assumed to realize the hierarchy: $m \gg 1/r \sim mv \gg \Lambda_{\rm QCD}$
At $mv \gg \mu \gg mv^2$ the EFT is weakly coupled pNRQCD; its degrees of freedom are

- Q-$\bar{Q}$ (singlet and octet): $E \sim \Lambda_{\rm QCD}, mv^2$; $p \lesssim mv$
- Gluons: $E \sim p \sim \Lambda_{\rm QCD}, mv^2$

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The J/ψ mass at $\mathcal{O}(m\alpha_{\rm s}^5)$ is

$$E_{J/\psi} = \langle J/\psi | H_s(\mu) | J/\psi \rangle - i \frac{g^2}{3N_c} \int_0^\infty dt \, \langle 1S | \mathbf{r} e^{it(E^{(0)}_{J/\psi} - H_o)} \mathbf{r} | 1S \rangle \, \langle \mathbf{E}(t) \, \mathbf{E}(0) \rangle (\mu)$$

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From which it follows

$$\bar{m}_c(m_c) = 1.24 \pm 0.020 \text{ GeV}$$

Brambilla Sumino Vairo 01

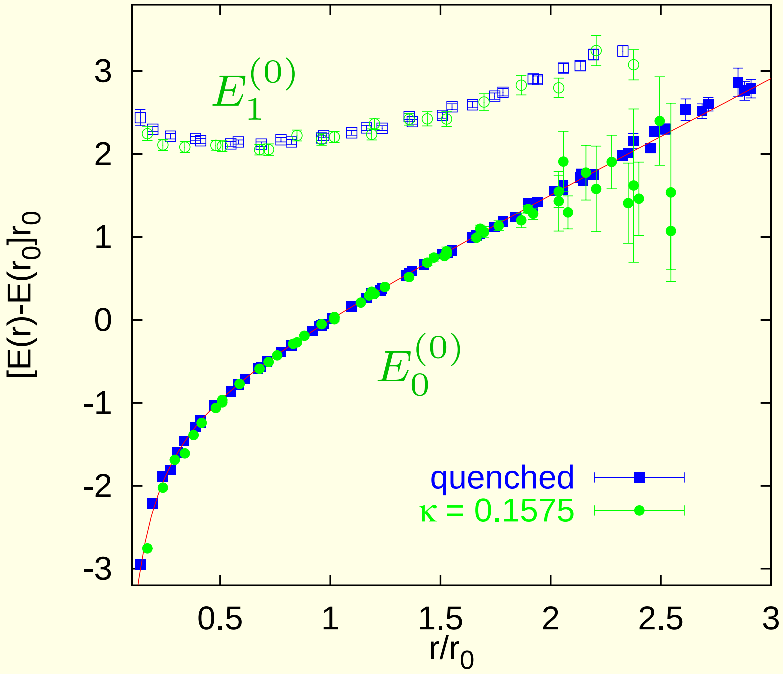
Higher resonances

Higher $c\bar{c}$ resonances are better studied on the lattice.

- QCD ($ma \ll 1$)
- NRQCD (coarse lattices, $ma \gg 1$, no $a \to 0$)
- pNRQCD (coarse lattices, no $a \to 0$)

pNRQCD for higher resonances

- All quarks with energy $\gg mv^2$ and momentum $\gg mv$ are integrated out.
- All gluonic excitations between heavy quarks are integrated out since they develop a gap of order $\Lambda_{\rm QCD}$ with the static $Q\bar{Q}$ energy.



Bali et al. 98

($r_0 \simeq 0.5\,$fm)

$\Rightarrow$ The singlet quarkonium field S of energy mv^2 and momentum mv is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

pNRQCD for higher resonances

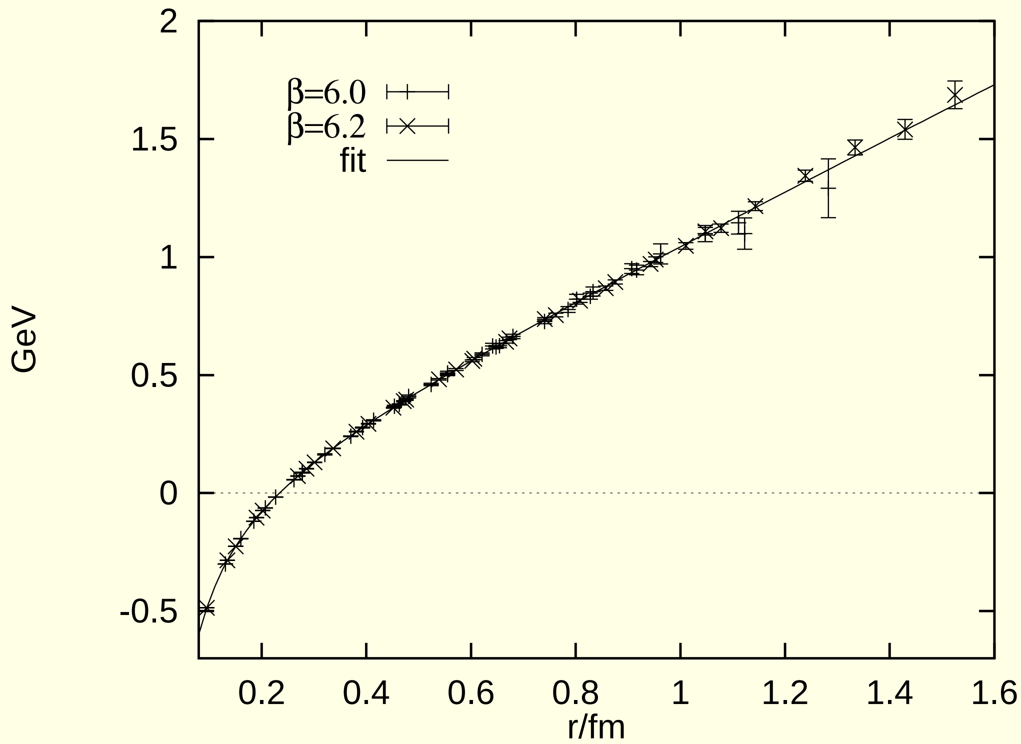
- All quarks with energy $\gg mv^2$ and momentum $\gg mv$ are integrated out.
- All gluonic excitations between heavy quarks are integrated out since they develop a gap of order $\Lambda_{\rm QCD}$ with the static $Q\bar{Q}$ energy.

$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\}$$

- The idea is to calculate once for ever the potentials on the lattice and determine the spectrum by solving the Schrödinger equation.

Static potential

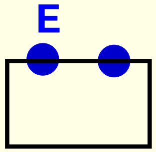
$$V_s^{(0)} = \lim_{T\to\infty} \frac{i}{T} \ln\langle W(r\times T)\rangle = \lim_{T\to\infty} \frac{i}{T} \ln \left\langle \Box \right\rangle$$



Bali Schilling Wachter 97

$1/m$ potential

$$V_s^{(1)} = -\frac{1}{2}\int_0^\infty dt\, t \left\langle \quad \right\rangle$$



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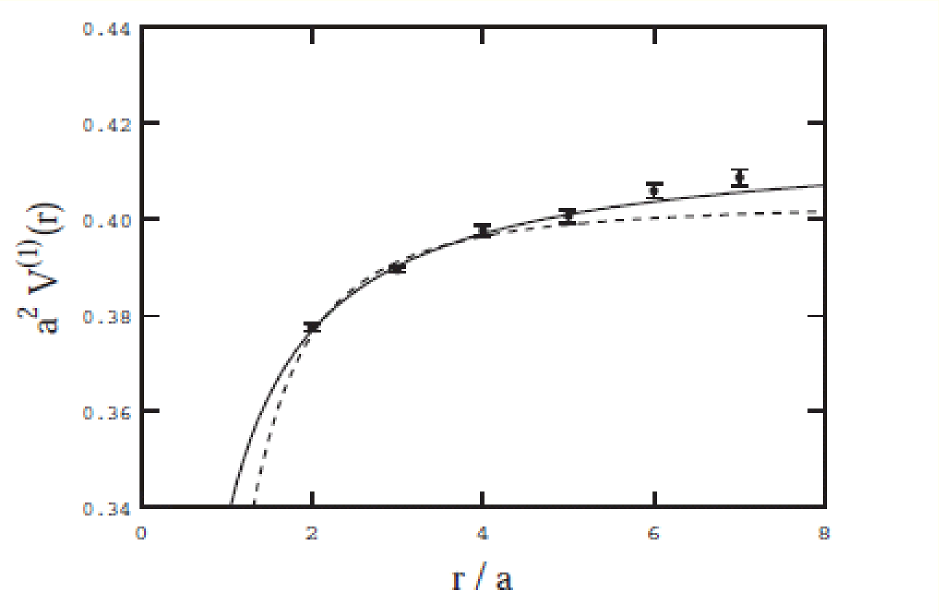
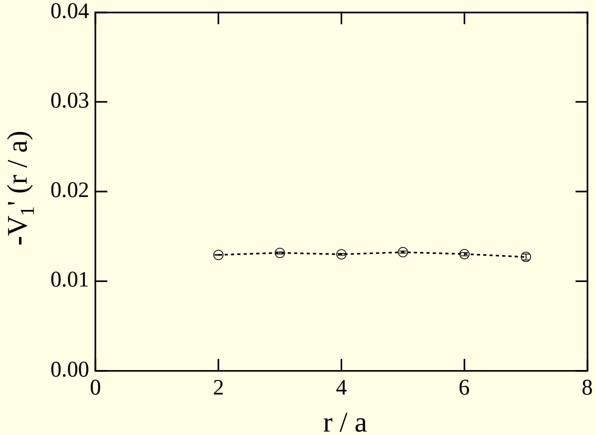


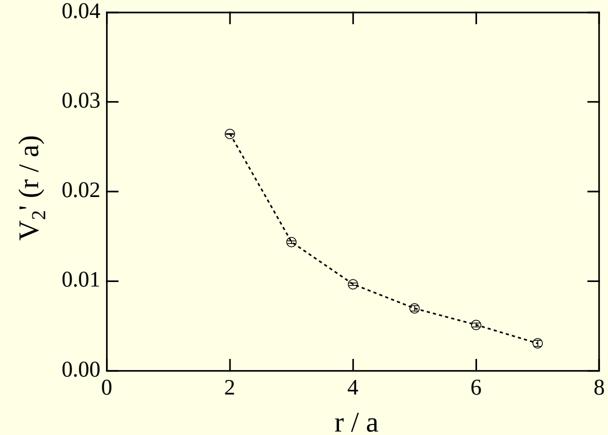
Koma, Koma and Wittig/QWG 06

Spin-dependent potentials

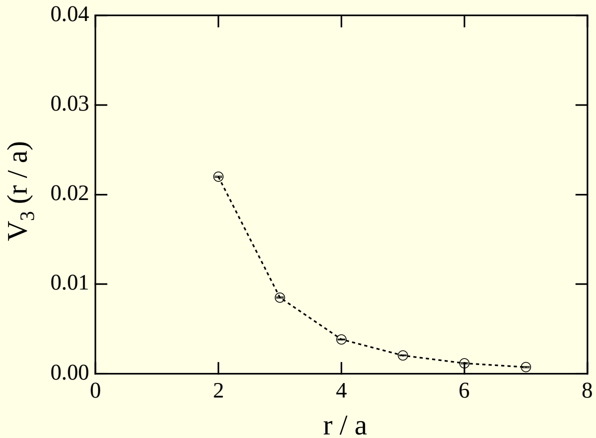
$$2\int_0^\infty d\tau\ \tau\langle\langle B_y(\mathbf{r},0)E_z(\mathbf{r},\tau)\rangle\rangle$$



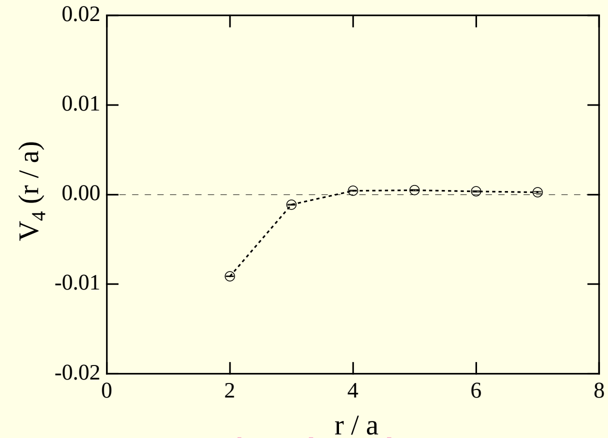
$$2\int_0^\infty d\tau\ \tau\langle\langle B_y(\mathbf{0},0)E_z(\mathbf{r},\tau)\rangle\rangle$$



$$2\int_0^\infty d\tau\ [\langle\langle B_x(\mathbf{0},0)B_x(\mathbf{r},\tau)\rangle\rangle - (x \to y)]$$



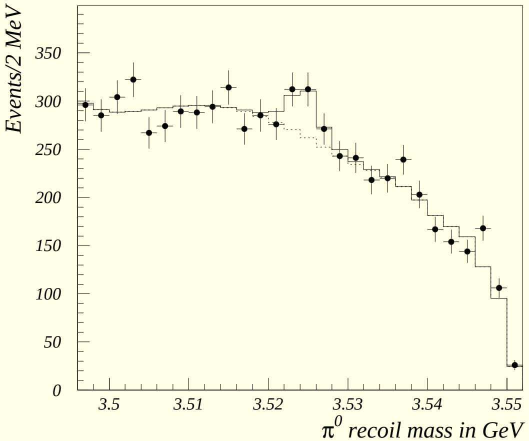
$$2\int_0^\infty d\tau\ [\langle\langle B_x(\mathbf{0},0)B_x(\mathbf{r},\tau)\rangle\rangle + 2(x \to y)]$$



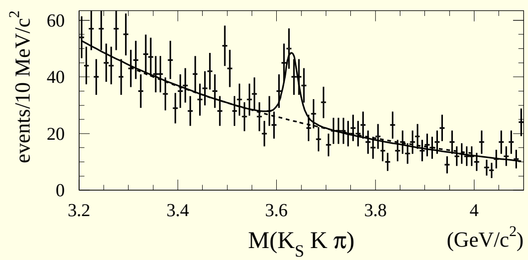
Koma Koma and Wittig 05, 06

States near or above threshold

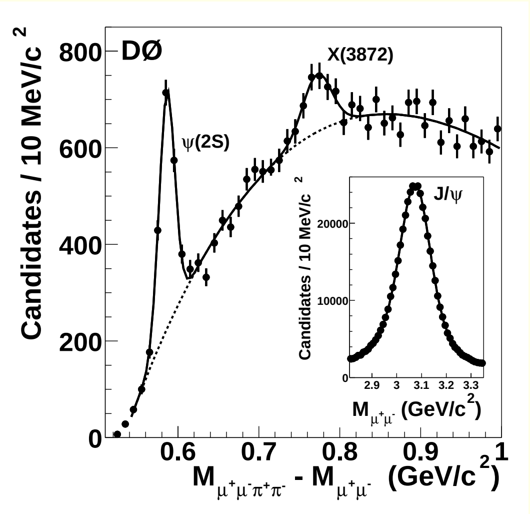
- In general, for states near or above threshold a systematic treatment does not exist so far. Most of the existing analyses rely on models (e.g. the Cornell coupled channel model).
- However one may still exploit an expansion in $\alpha_{\rm s}$ and v. In some cases one may develop an EFT owing to special dynamical conditions.
  - A possible exotic (hybrid) is the $Y(4260)$.
  - An example is the $X(3872)$ intepreted as a $D^0\,\bar{D}^{*\,0}$ or $\bar{D}^0\,D^{*\,0}$ molecule. In this case, one may take advantage of the unnaturally (and accidentally) large $D^0\,\bar{D}^{*\,0}$ scattering length. Braaten Kusunoki 03



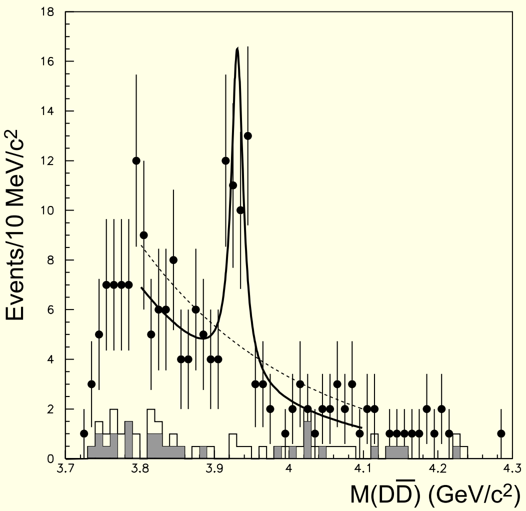
$h_c(3523)$ CLEO 05
E835 05



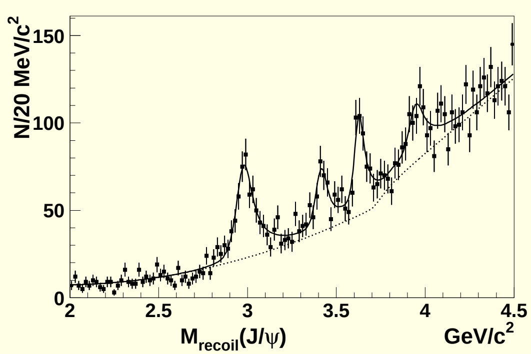
$\eta_c(2S)(3630)$ BaBar 04
CLEO 04
Belle 02



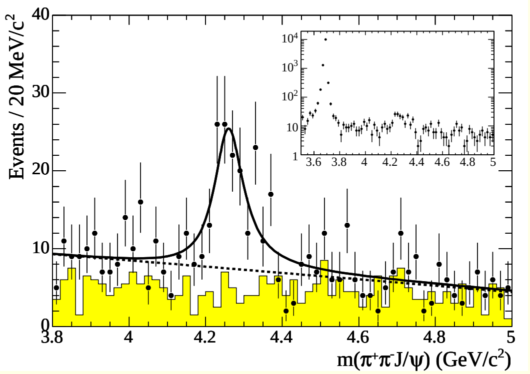
X(3872) CDF D0/QWG 04
Belle 02
BaBar 05



Z(3930) Belle 05



X(3940) Belle 05



Y(4260) BaBar 05

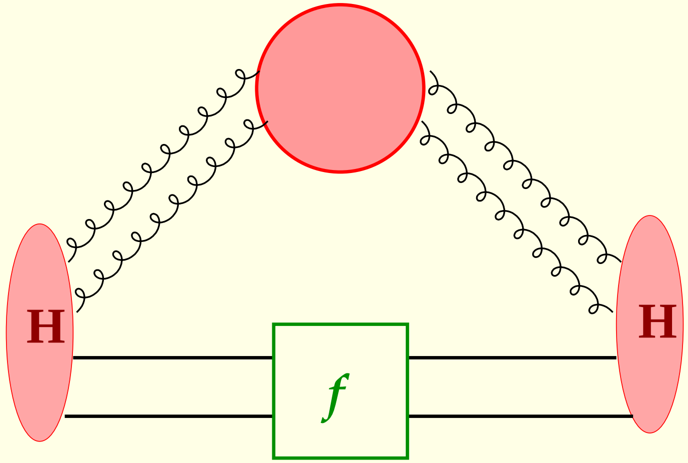
2. Annihilations

NRQCD factorization

$$\Gamma(H \to \mathrm{LH}) = \sum_n \frac{2\,\mathrm{Im}\, f^{(n)}}{m^{d_n-4}} \langle H|\psi^\dagger K^{(n)}\chi\chi^\dagger K'^{\,(n)}\psi|H\rangle$$

$$\Gamma(H \to \mathrm{EM}) = \sum_n \frac{2\,\mathrm{Im}\, f^{(n)}_{\mathrm{em}}}{m^{d_n-4}} \langle H|\psi^\dagger K^{(n)}\chi|\mathrm{vac}\rangle\langle \mathrm{vac}|\chi^\dagger K'^{\,(n)}\psi|H\rangle$$

Bodwin et al 95



NRQCD matrix elements

- By fitting charmonium P-wave decay data
$\langle O_1(^1P_1)\rangle_{h_c(1P)} \approx 8.1 \times 10^{-2}$ GeV^5 and $\langle O_8(^1S_0)\rangle_{h_c(1P)} \approx 5.3 \times 10^{-3}$ GeV^3
in $\overline{\mathrm{MS}}$ and at the factorization scale of 1.5 GeV.

Maltoni 00

- In quenched lattice simulations
$\langle O_1(^1P_1)\rangle_{h_c(1P)} \approx 8.0 \times 10^{-2}$ GeV^5 , $\langle O_8(^1S_0)\rangle_{h_c(1P)} \approx 4.7 \times 10^{-3}$ GeV^3 and
$\langle O_1(^1S_0)\rangle_{\eta_c(1S)} \approx 0.33$ GeV^3
in $\overline{\mathrm{MS}}$ and at the factorization scale of 1.3 GeV.

Bodwin Sinclair Kim 96

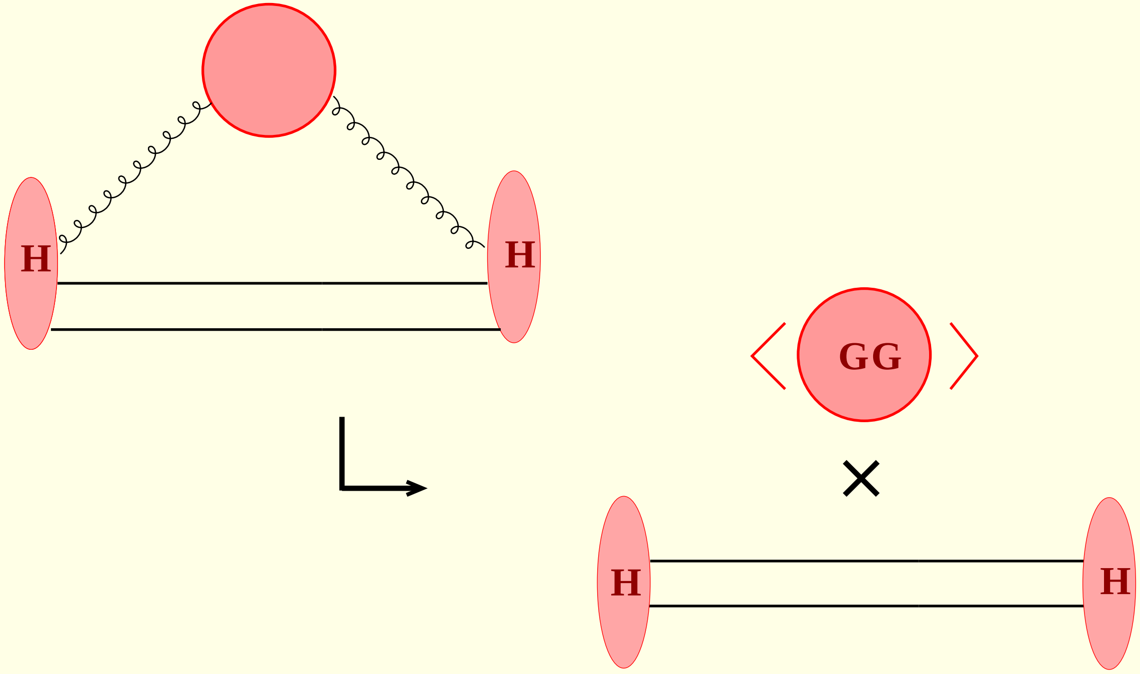
- In lattice simulations with three light-quark flavors (extrapolation)
$\langle O_1(^1S_0)\rangle_{\eta_b(1S)} \approx 4.1$ GeV^3, $\langle O_1(^1P_1)\rangle_{h_b(1P)} \approx 3.3$ GeV^5 and
$\langle O_8(^1S_0)\rangle_{h_b(1P)} \approx 5.9 \times 10^{-3}$ GeV^3
in $\overline{\mathrm{MS}}$ and at the factorization scale of 4.3 GeV.

Bodwin Sinclair Kim 01

Some further recent (quenched) determinations are in Bodwin Lee Sinclair 05

pNRQCD factorization

$$\langle H|\psi^\dagger K^{(n)}\chi\chi^\dagger K'^{(n)}\psi|H\rangle = |R(0)|^2 \times \int dt\, t^n \,\langle G(t)G(0)\rangle$$



Brambilla Eiras Pineda Soto Vairo 02 03

P-wave decays at $\mathcal{O}(mv^5)$

- NRQCD

$$\Gamma(\chi_J \to \mathrm{LH}) \;=\; 9\ \mathrm{Im}\, f_1\, \frac{\left|R'(0)\right|^2}{\pi m^4} \;+\; \frac{2\ \mathrm{Im}\, f_8}{m^2}\, \langle \chi | O_8(^1S_0) | \chi \rangle$$

$$\Gamma(\chi_J \to \gamma\gamma) \;=\; 9\ \mathrm{Im}\, f_{\gamma\gamma}\, \frac{\left|R'(0)\right|^2}{\pi m^4} \qquad J = 0, 2$$

* *Bottomonium and charmonium P-wave decays depend on 6 non-perturbative parameters.*

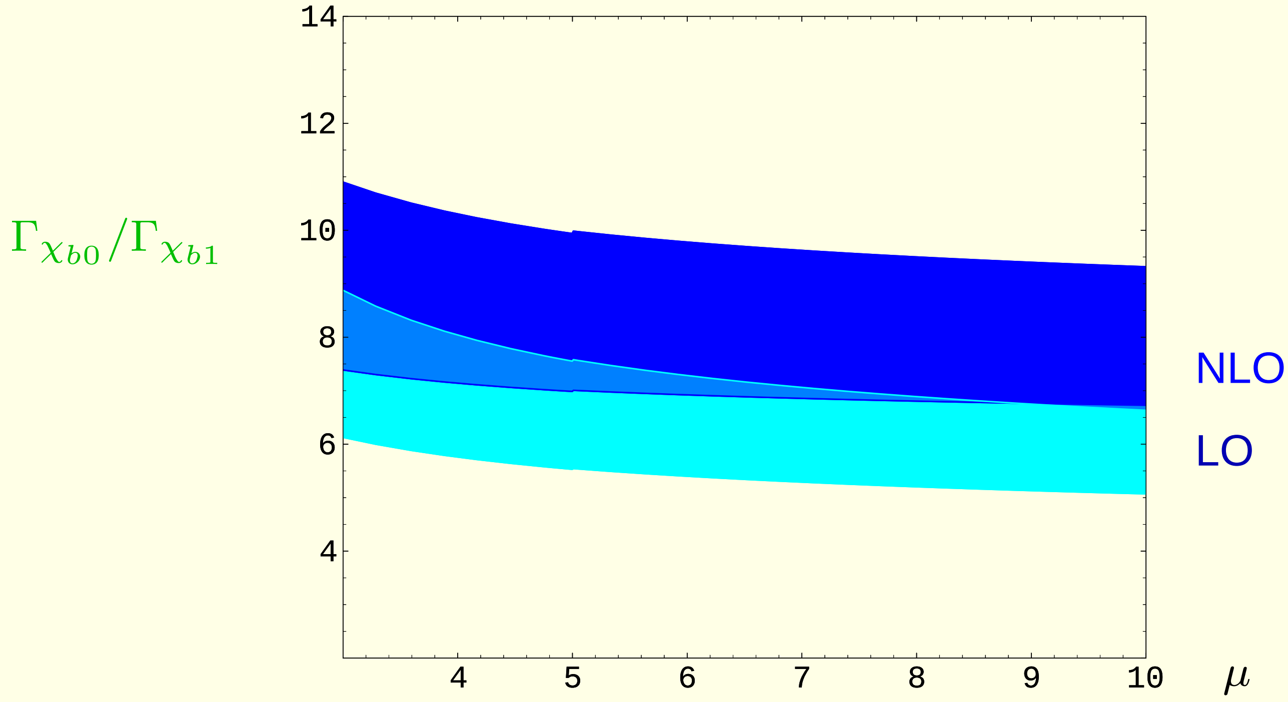
- pNRQCD

$$\langle \chi | O_8(^1S_0) | \chi \rangle = \frac{\left|R'(0)\right|^2}{18\pi m^2}\, \mathcal{E}; \quad \mathcal{E} \equiv \int_0^\infty dt\, t^3\, \langle \mathrm{Tr}(g\mathbf{E}(t)\, g\mathbf{E}(0)) \rangle$$

* *The quarkonium state dependence factorizes.*

* *Bottomonium and charmonium P-wave decays depend on 4 non-perturbative parameters.*

Bottomonium P-wave decays



$$\frac{\Gamma(\chi_{b0}(1P) \to \mathrm{LH})}{\Gamma(\chi_{b1}(1P) \to \mathrm{LH})} = \frac{\Gamma(\chi_{b0}(2P) \to \mathrm{LH})}{\Gamma(\chi_{b1}(2P) \to \mathrm{LH})} = 8.0 \pm 1.3$$

(CleoIII 02) $= 19.3 \pm 9.8$

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Charmonium P-wave decays

Ratio	PDG04	PDG00	LO	NLO
$\frac{\Gamma(\chi_{c0}\to\gamma\gamma)}{\Gamma(\chi_{c2}\to\gamma\gamma)}$	5.1±1.1	13±10	3.75	$\approx$ 5.43
$\frac{\Gamma(\chi_{c2}\to l.h.)-\Gamma(\chi_{c1}\to l.h.)}{\Gamma(\chi_{c0}\to\gamma\gamma)}$	410±100	270±200	$\approx$ 347	$\approx$ 383
$\frac{\Gamma(\chi_{c0}\to l.h.)-\Gamma(\chi_{c1}\to l.h.)}{\Gamma(\chi_{c0}\to\gamma\gamma)}$	3600±700	3500±2500	$\approx$ 1300	$\approx$ 2781
$\frac{\Gamma(\chi_{c0}\to l.h.)-\Gamma(\chi_{c2}\to l.h.)}{\Gamma(\chi_{c2}\to l.h.)-\Gamma(\chi_{c1}\to l.h.)}$	7.9±1.5	12.1±3.2	2.75	$\approx$ 6.63
$\frac{\Gamma(\chi_{c0}\to l.h.)-\Gamma(\chi_{c1}\to l.h.)}{\Gamma(\chi_{c2}\to l.h.)-\Gamma(\chi_{c1}\to l.h.)}$	8.9±1.1	13.1±3.3	3.75	$\approx$ 7.63

$m_c = 1.5$ GeV $\quad \alpha_s(2m_c) = 0.245$

mainly from E835 (χ_{c0}, total width and $\gamma\gamma$)

also from Belle ($\chi_{c0}\to\gamma\gamma$) and CLEO, BES

QWG CERN report 04

$\Gamma(\eta_c \to LH)/\Gamma(\eta_c \to \gamma\,\gamma)$

- Large $\beta_0 \alpha_\mathrm{s}$ contributions.

$$\frac{\Gamma(\eta_c \to LH)}{\Gamma(\eta_c \to \gamma\,\gamma)} \approx (1.1\ \text{(LO)} + 1.0\ \text{(NLO)}) \times 10^3 = 2.1 \times 10^3$$

$$\frac{\Gamma(\eta_c \to LH)}{\Gamma(\eta_c \to \gamma\,\gamma)} = (3.3 \pm 1.3) \times 10^3\ \text{(EXP)}$$

$\Gamma(\eta_c \to LH)/\Gamma(\eta_c \to \gamma\,\gamma)$

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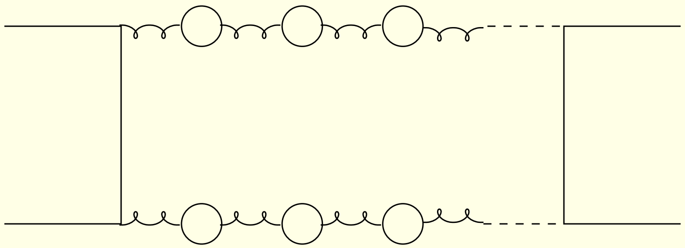
$$\frac{\Gamma(\eta_c \to LH)}{\Gamma(\eta_c \to \gamma\,\gamma)} = (3.3 \pm 1.3) \times 10^3 \text{ (EXP)}$$

- scheme dependence
- renormalons

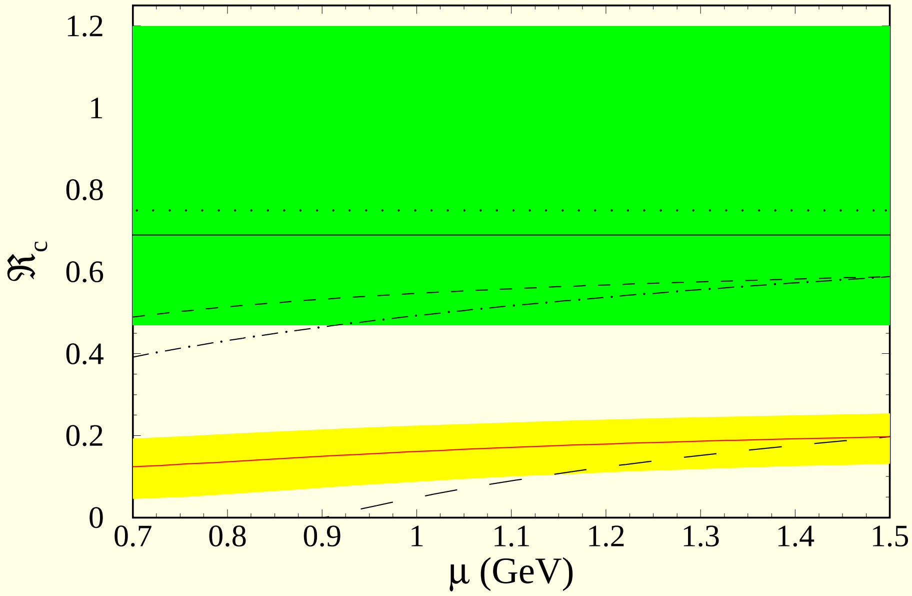


$$\frac{\Gamma(\eta_c \to LH)}{\Gamma(\eta_c \to \gamma\,\gamma)} = (3.01 \pm 0.30 \pm 0.34) \times 10^3$$

Bodwin Chen 01

$$\Gamma(J/\psi \to e^+e^-)/\Gamma(\eta_c \to \gamma\gamma)$$

- Large logarithms.



LO . . .

NLO _ _ _

NNLO __ __ __

NLL _ . _ .

NNLL _____

$$\mathcal{R}_c = \frac{\Gamma(J/\psi \to e^+e^-)}{\Gamma(\eta_c \to \gamma\gamma)}$$

Penin Pineda Smirnov Steinhauser 04

3. Production

Quarkonium Production

- There is no formal proof of the NRQCD factorization yet.
- The relevant 4-fermion operators are

$$\psi^\dagger K^{(n)} \chi \, a_H^\dagger a_H \, \chi^\dagger K'^{(n)} \psi$$

Recently it has been proved that the cancellation of the IR divergences at NNLO requires the modification of the 4 fermion operators into

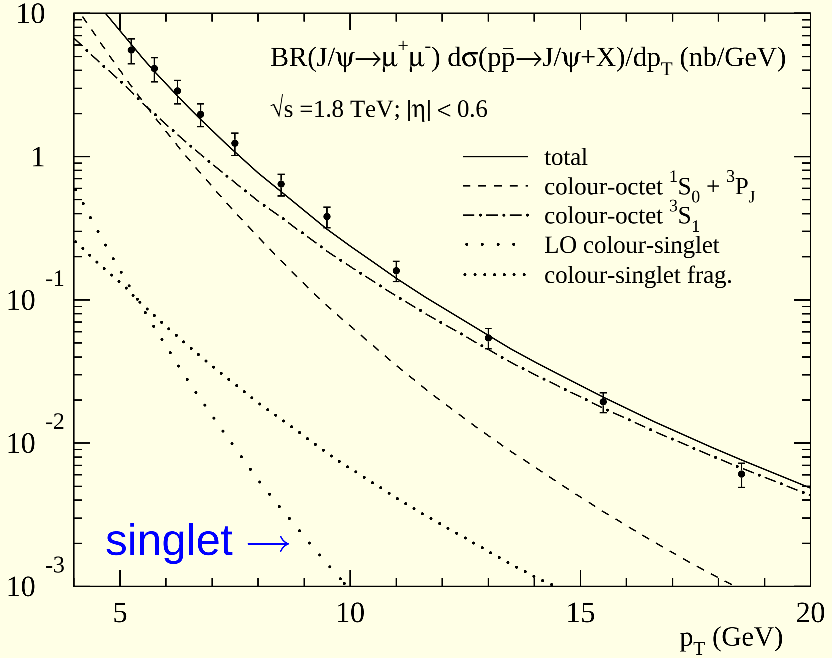
$$\psi^\dagger K^{(n)} \chi \, \phi_l^\dagger(0,\infty) \, a_H^\dagger a_H \, \phi_l(0,\infty) \, \chi^\dagger K'^{(n)} \psi$$

$$\phi_l(0,\infty) = \mathrm{P} \, \exp\left(-ig \int_0^\infty d\lambda \, l \cdot A(\lambda \, l)\right), \qquad l^2 = 1$$

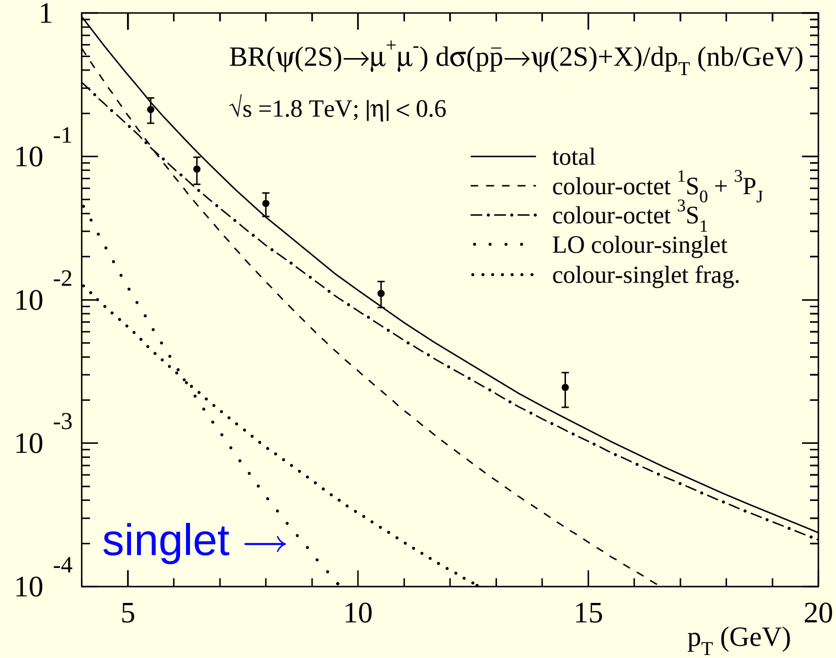
Nayak Qiu Sterman 05, Nayak/QWG 06

Charmonium Production at the Tevatron

Octet contributions dominate in production at high p_T.



$p\bar{p} \to J/\psi + X$



$p\bar{p} \to \psi(2S) + X$

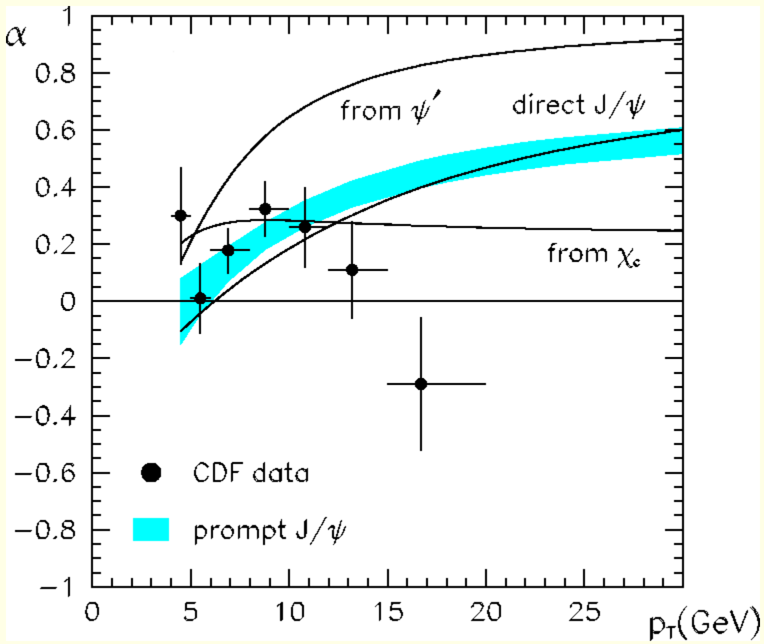
Krämer 01, CDF 97

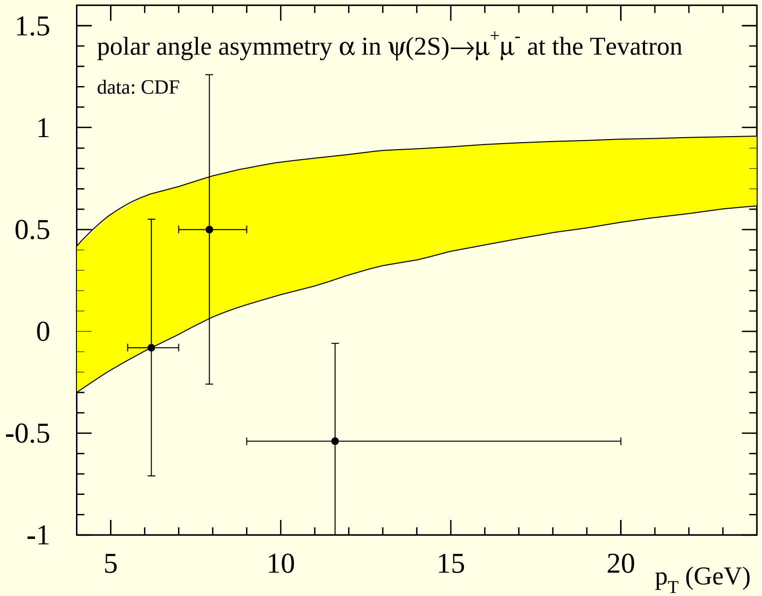
Charmonium Polarization at the Tevatron

- For large p_T quarkonium production, gluon fragmentation via the color-octet mechanism dominates: $\langle O_8^{J/\psi}(^3S_1)\rangle$.
- At large p_T the gluon is nearly on mass shell and so is transversely polarized.
- In color octet gluon fragmentation, most of the gluon's polarization is transferred to the J/ψ.
- Radiative corretions, color singlet production dilute this.
- In the case of the J/ψ feeddown is important:
  feeddown from χ_c states is about 30% of the J/ψ sample and dilutes the polarization.
- feeddown from $\psi(2S)$ is about 10% of the J/ψ sample and is largely transversely polarized.

- *Spin-flippling terms are assumed suppressed. But This stricly depends on the power counting.*
  If they are not, polarization may dilute at high p_T.

Charmonium Polarization at the Tevatron



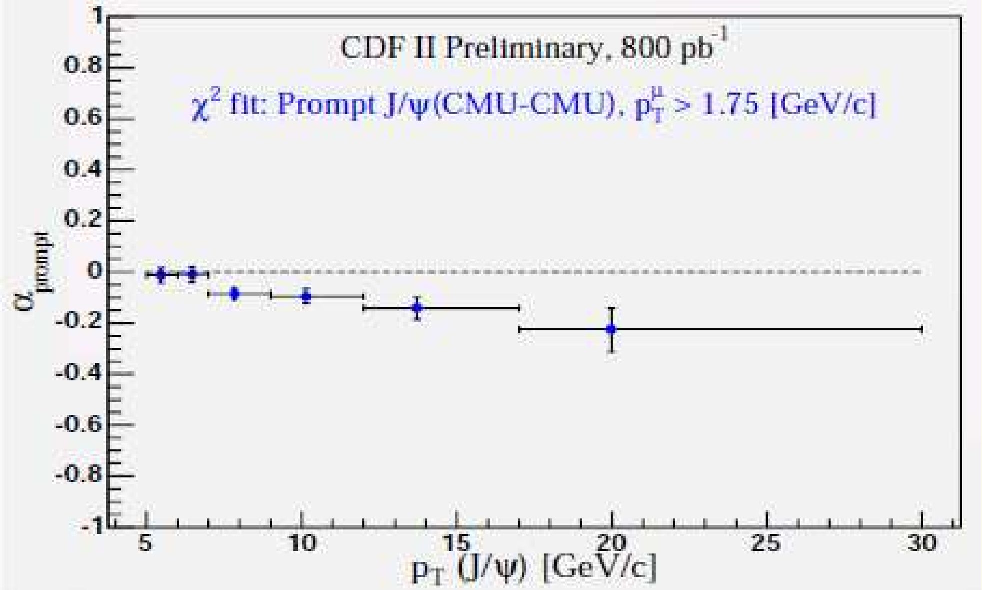


$$\frac{d\sigma}{d\cos\theta} \propto 1 + \alpha \cos^2\theta$$

$\alpha = 1$ is completely transverse $\qquad \alpha = -1$ is completely longitudinal.

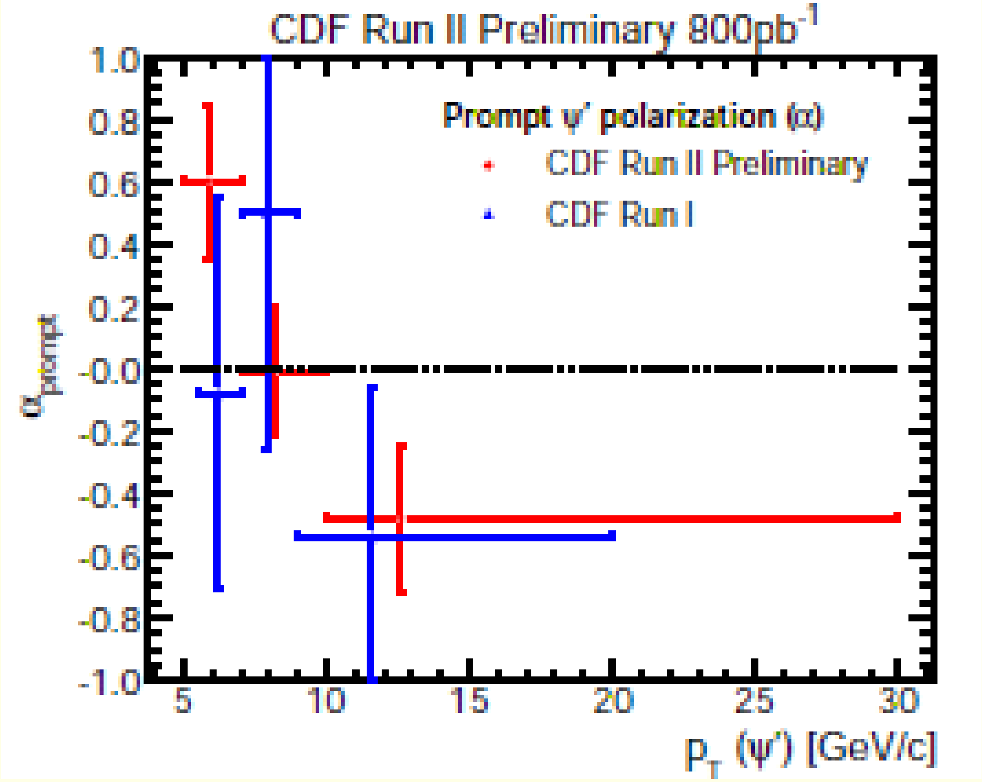
Krämer 01, Braaten et al 01, CDF 97

Charmonium Polarization at the Tevatron



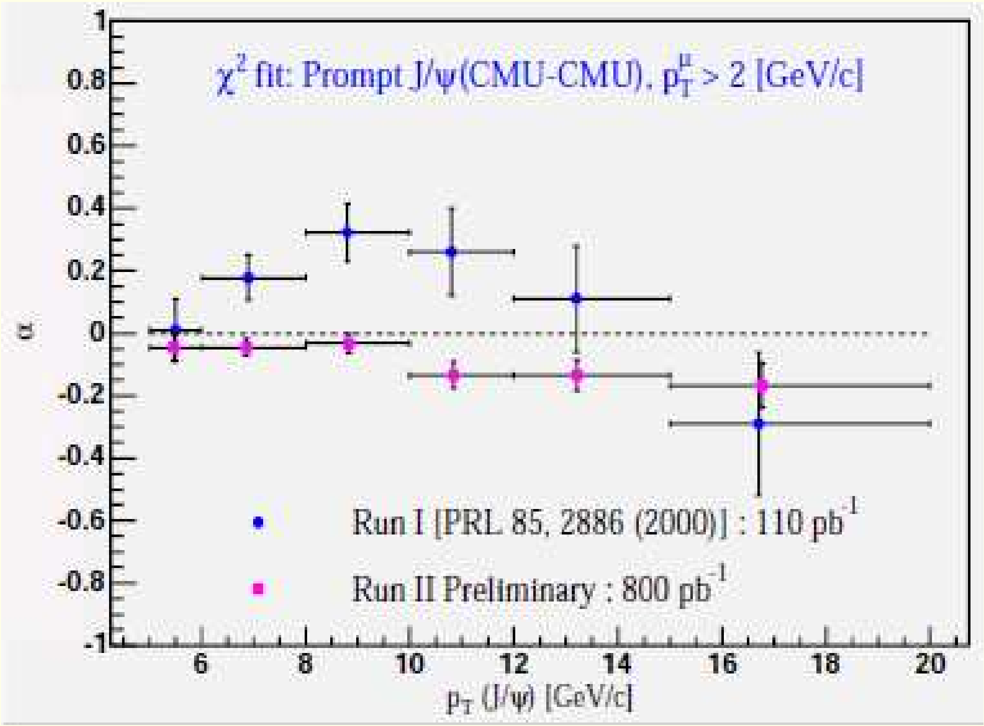
CDF/QWG 06

Charmonium Polarization at the Tevatron



CDF/QWG 06

Charmonium Polarization at the Tevatron



CDF/QWG 06

5. Prospects of $p\bar{p}$ at Fermilab

E760-E835 legacy

$p\bar{p}$ to charmonium is good for

- precise determination of known resonances
  * mass measurements at about 0.1 MeV level,
  * total width measurements (ideal for narrow states);
- through the detection of:
  EM final states (e.g. electrons and photons) few body final states.

Mussa/QWG 06

E760-E835 legacy

$p\bar{p}$ to charmonium was limited by

- non hermicity of the detector;
- low energy photon threshold: 20 MeV;
- calorimeter granularity;
- multiple scattering for tracks below 1 GeV;
- physical occupancy of the jet target;
- 2x extra rate induced by e-$\bar{p}$ interactions;
- no momentum measurement on hadrons (no magnet).

The current generation of B factories will not have enough statistics to measure $p\bar{p}$ coupling to newly discovered states. Probably super B factories will be able to measure some in B decays (limited to J=0,1). Panda is scheduled to start after 2014, i.e. in the super B era.

The antiproton source at Fermilab (the antiproton ring should have originally supplied beam to BTeV) provides (after 2009 and before 2014: data taking from 2011) a potential tool to understand XYZ's. What is needed is:

- an improved detector and a target with respect to the old E835;
- B factories that tell where to look.

Gollwitzer Mussa/QWG 06

Conclusion

Charmonium physics provides a place where low-energy QCD and its rich structure may be studied in a controlled and systematic way, combining perturbative QCD, lattice calculations, and effective field theory analytical methods.

B factories have immensely boosted charmonium studies in the last years, which makes the contribution to charmonium physics that $p\bar{p}$ machines at Fermilab and GSI have provided and could/will provide in the future even more valuable.