Radiative transitions and the quarkonium magnetic moment

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based on

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Model-independent study of magnetic dipole transitions in quarkonium

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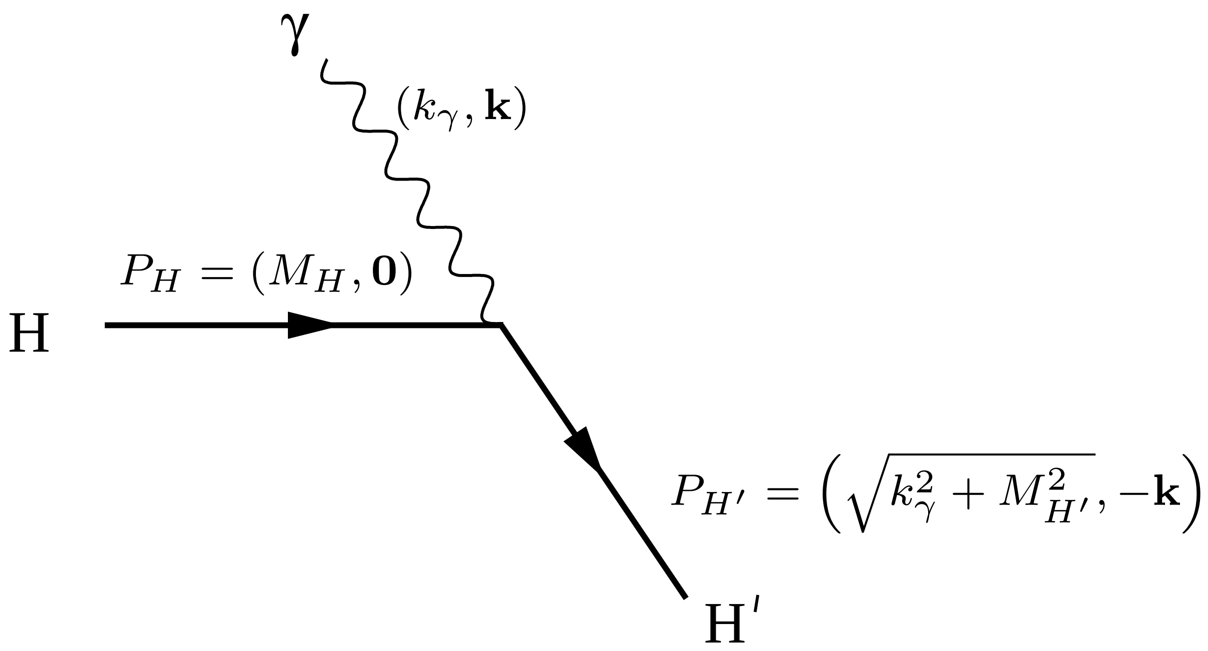
Radiative transitions: basics

Two dominant single-photon-transition processes:

(1) electric dipole transitions (E1)

(2) magnetic dipole transitions (M1)



Radiative transitions: basics

Two dominant single-photon-transition processes:
(1) electric dipole transitions (E1)
(2) magnetic dipole transitions (M1)

In the non-relativistic limit

$$\Gamma^{\mathrm{M1}}_{n^3S_1 \to n'^1S_0\,\gamma} = \frac{4}{3}\,\alpha\,e_Q^2\,\frac{k_\gamma^3}{m^2}\,\left|\int_0^\infty dr\,r^2\,R_{n'0}(r)\,R_{n0}(r)\,j_0\left(\frac{k_\gamma r}{2}\right)\right|^2$$

If $k_\gamma\langle r\rangle \ll 1$ $\quad j_0(k_\gamma r/2) = 1 - (k_\gamma r)^2/24 + \ldots$

- $n = n'$ allowed transitions
- $n \neq n'$ hindered transitions

$J/\psi \to \eta_c \gamma$

Only one direct experimental measure:

$$\Gamma(J/\psi \to \eta_c\gamma) = (1.14 \pm 0.23)\,\text{keV} \qquad \text{Crystal Ball 86}$$

Moreover, there are several measurements of the BR $J/\psi \to \eta_c\gamma \to \phi\phi\gamma$ and one independent measurement of $\eta_c \to \phi\phi$ (Belle 03). From them one obtains

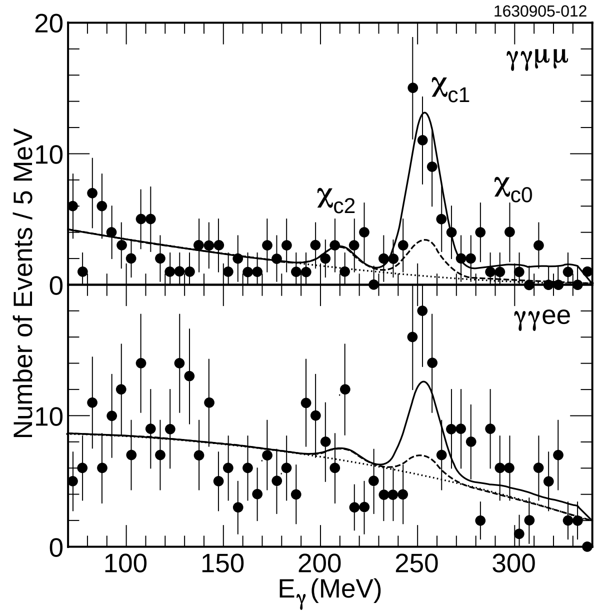
$$\Gamma(J/\psi \to \eta_c\gamma) = (2.9 \pm 1.5)\,\text{keV}$$

Combining both

$$\Gamma(J/\psi \to \eta_c\gamma) = (1.18 \pm 0.36)\,\text{keV} \qquad \text{PDG 04}$$

- $\Gamma(J/\psi \to \eta_c\gamma)$ enters into many charmonium BR.
  Its 30% uncertainty sets typically their experimental errors.

$\psi(3770) \to \chi_{c1}\gamma$



First resolved radiative transition from a D-wave state:

$$\mathcal{B}(\psi(3770) \to \chi_{c1}\gamma) = (3.2 \pm 0.6 \pm 0.4) \times 10^{-3}$$

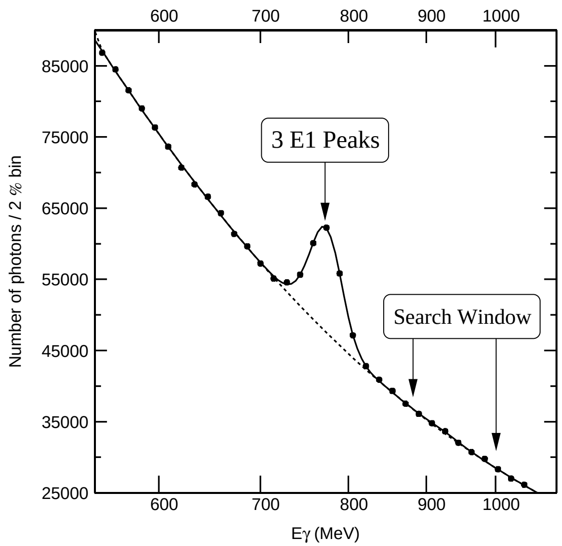
CLEO 05

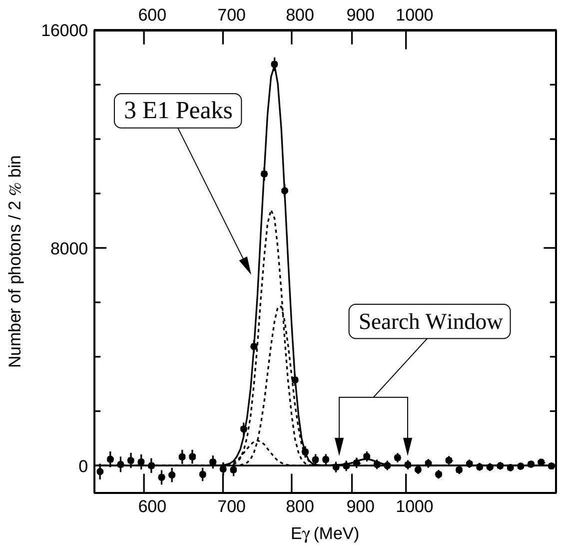
- $\Upsilon(1D)$ transitions have been observed in the cascade:

$$\Upsilon(3S) \to \chi_b(2P)\gamma, \quad \chi_b(2P) \to \Upsilon(1D)\,\gamma, \quad \Upsilon(1D) \to \chi_b(1P)\gamma, \quad \chi_b(1P) \to \Upsilon(1S)\,\gamma$$

CLEO 04

η_b search

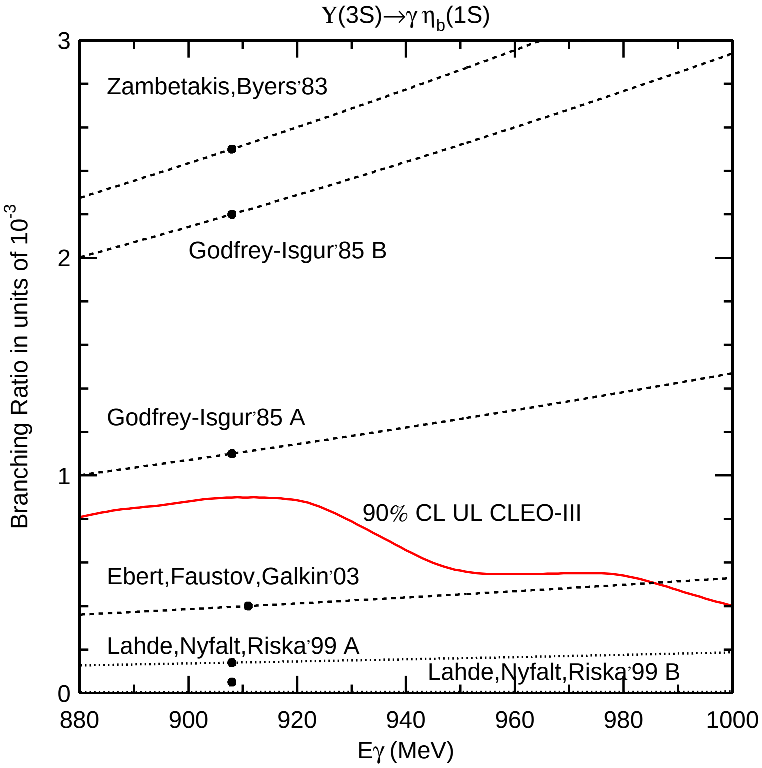




- Search through $\Upsilon(3S) \rightarrow \eta_b(1S)\,\gamma$ (M1 hindered transition)
  Potentially promising due to high γ energy (k)
  - better resolution
  - $\Gamma \propto k^3$
- Other (observed) transitions $\Upsilon(3S) \rightarrow \chi_b(2P_J)\,\gamma \rightarrow \Upsilon(1S)\,\gamma$

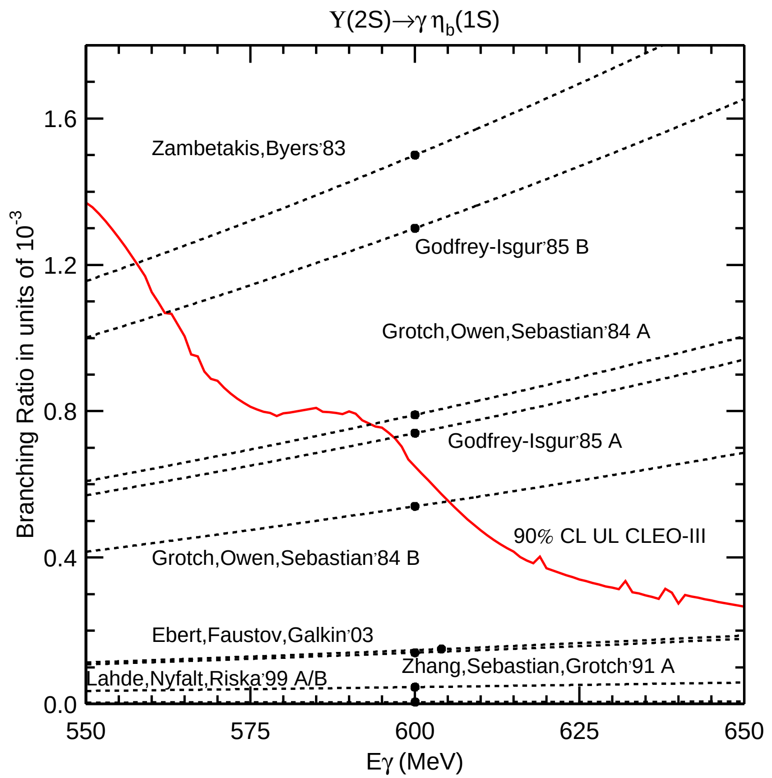
CLEO 02

$$\Upsilon(3S) \to \eta_b(1S)\gamma$$



CLEO/QWG 04

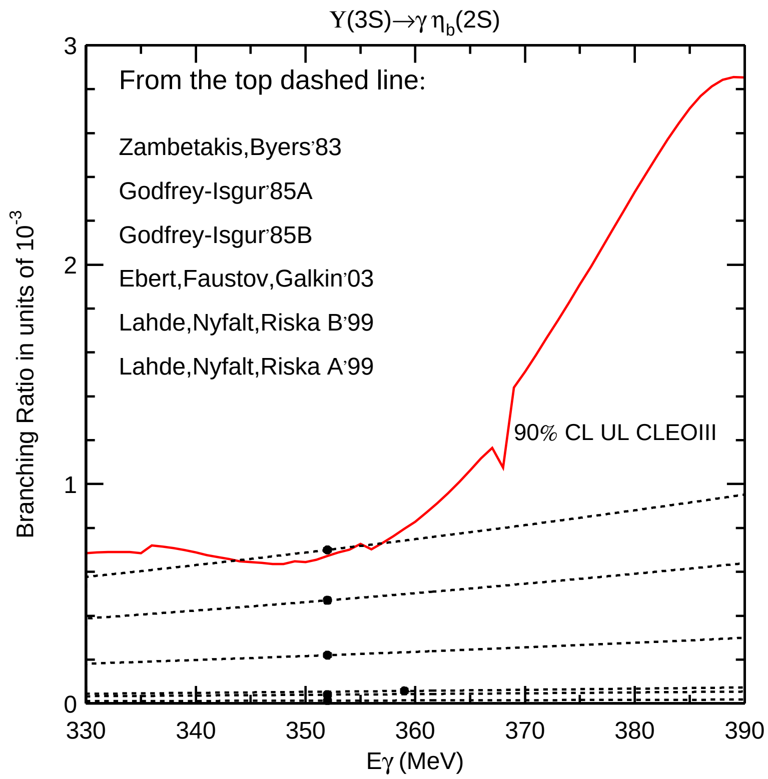
$$\Upsilon(2S) \to \eta_b(1S)\gamma$$



The (non) observed transition rates are becoming problematic for most models.

CLEO/QWG 04

$$\Upsilon(3S) \to \eta_b(2S)\gamma$$



CLEO/QWG 04

Several model determinations exist.

Grotch Owen Sebastian 84

Several model determinations exist.

Grotch Owen Sebastian 84

- Relativistic equation with scalar and vector potentials.
- Non-relativistic reduction.
- Relativistic invariance is somewhat imposed to calculate recoil corrections.

We would like to understand

- to which extent these determinations are consistent with QCD;
- what is their range of applicability.

In practice, we would like to have theoretical determinations

- with an error bar attached to them;
- improvable in a systematic way.

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These are provided by Effective Field Theories.

Scales

- $p \sim \frac{1}{r} \sim mv, \qquad E \sim mv^2$
- $\Lambda_{\rm QCD}$
- k_γ

In a non-relativistic system $mv \gg mv^2$.

$k_\gamma \sim mv^2$ for hindered transitions;
$k_\gamma \sim mv^4$ for allowed transitions.
As a consequence $k_\gamma r \ll 1$.

Degrees of freedom

- Degrees of freedom at scales lower than mv:

  Q-$\bar{Q}$ states, with energy $\sim \Lambda_{\rm QCD}$, mv^2 and momentum $\lesssim mv$
  $\Rightarrow$ (i) singlet $\rm S$ (ii) octet $\rm O$ (if $mv \gg \Lambda_{\rm QCD}$)

  Gluons with energy and momentum $\sim \Lambda_{\rm QCD}$, mv^2 (if $mv \gg \Lambda_{\rm QCD}$)

  Photons of energy and momentum lower than mv.

- Power counting:

  $p \sim \frac{1}{r} \sim mv$;
  all gauge fields are multipole expanded: $A(R, r, t) = A(R, t) + \mathbf{r} \cdot \boldsymbol{\nabla} A(R, t) + \ldots$
  and scale like $(\Lambda_{\rm QCD}$ or $mv^2)^{\rm dimension}$.

Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\rm pNRQCD} \quad = \quad & -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} - \frac{1}{4}F^{\rm em}_{\mu\nu}F^{\mu\nu\,{\rm em}} \\
& + {\rm Tr}\left\{ {\rm S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) {\rm S} \right. \\
& \left. + \; {\rm O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) {\rm O} \right\}
\end{aligned}
$$

LO in r

$$
\begin{aligned}
& + {\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm S} + {\rm S}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm O} \right\} \\
& + \frac{1}{2}{\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm O} + {\rm O}^\dagger {\rm O}\mathbf{r}\cdot g\mathbf{E} \right\} \qquad (\text{if } mv \gg \Lambda_{\rm QCD}) \\
& + \cdots
\end{aligned}
$$

NLO in r

$$
+\mathcal{L}_\gamma
$$

Pineda Soto 97, Brambilla Pineda Soto Vairo 99

$\mathcal{L}_\gamma$

$$\begin{aligned}
\mathcal{L}_\gamma \quad = \quad & \mathrm{Tr}\Bigg\{ V_A^{\mathrm{em}}\, \mathrm{S}^\dagger \mathbf{r}\cdot ee_Q \mathbf{E}^{\mathrm{em}} \mathrm{S} \\
& + \frac{1}{2m}\, V_1\, \left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot ee_Q \mathbf{B}^{\mathrm{em}}\right\} \mathrm{S} \\
& + \frac{1}{2m}\, V_1\, \left\{\mathrm{O}^\dagger, \boldsymbol{\sigma}\cdot ee_Q \mathbf{B}^{\mathrm{em}}\right\} \mathrm{O} \qquad (\text{if } mv \gg \Lambda_{\mathrm{QCD}}) \\
& + \frac{1}{4m^2}\, \frac{V_2}{r}\, \left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot \left[\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times ee_Q \mathbf{B}^{\mathrm{em}}\right)\right]\right\} \mathrm{S} \\
& + \frac{1}{4m^2}\, \frac{V_3}{r}\, \left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot ee_Q \mathbf{B}^{\mathrm{em}}\right\} \mathrm{S} \\
& + \frac{1}{4m^3}\, V_4\, \left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot ee_Q \mathbf{B}^{\mathrm{em}}\right\} \boldsymbol{\nabla}_r^2 \mathrm{S} + \cdots \Bigg\}
\end{aligned}$$

Matching

The matching consists in the calculation of the coefficients V.
They get contributions from

- hard modes ($\sim m$):

$$\bar{\psi}(i\not{D} - m)\psi \to \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{c_F^{\rm em}}{2m}\boldsymbol{\sigma}\cdot ee_Q\mathbf{B}^{\rm em} + \cdots \right)\psi$$

  From HQET:

$$c_F^{\rm em} \equiv 1 + \kappa = 1 + 2\frac{\alpha_{\rm s}}{3\pi} + \mathcal{O}(\alpha_{\rm s}^2)$$

  is the quark magnetic moment.

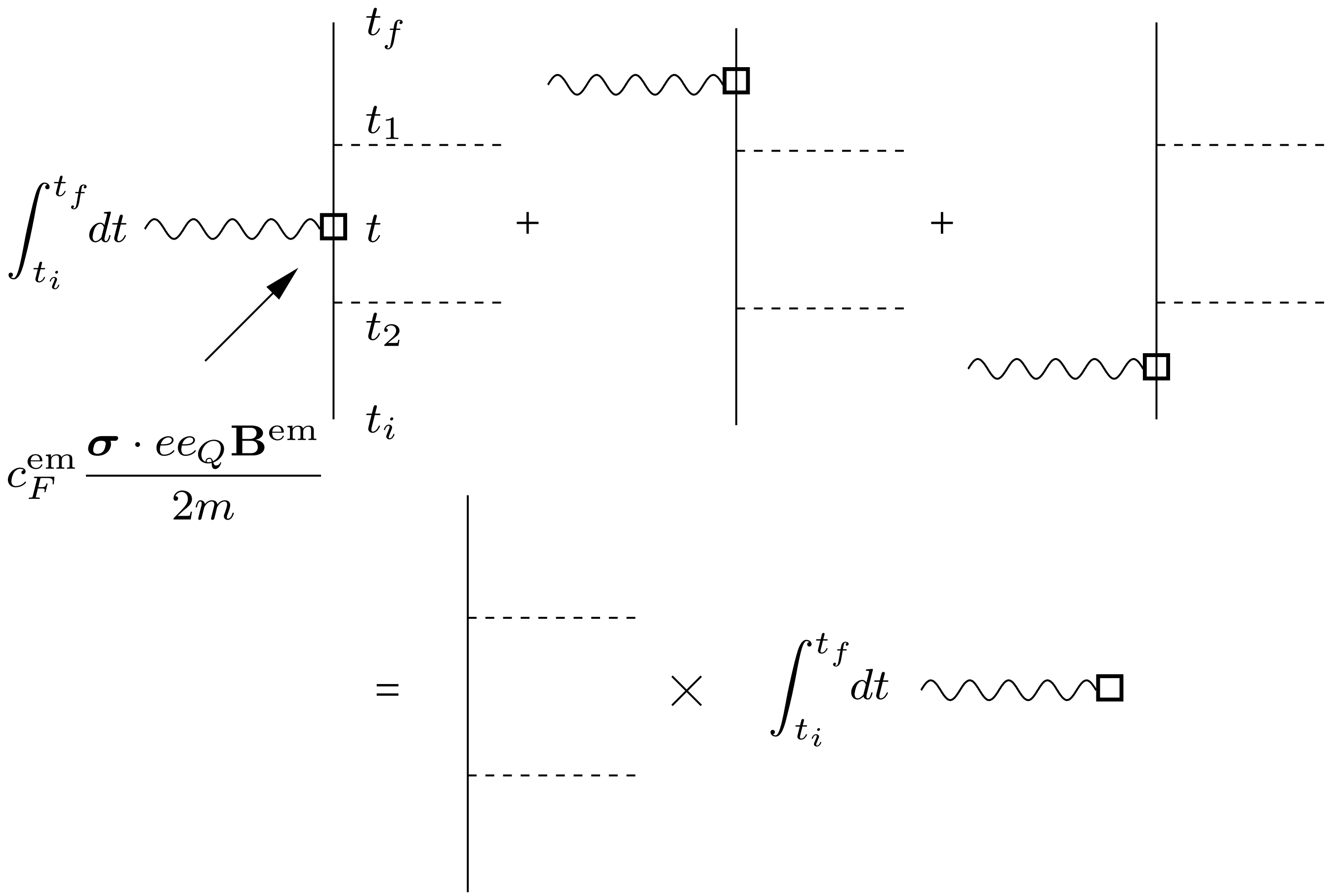
- soft modes ($\sim mv$).

M1 operator at $\mathcal{O}(1)$

$$V_1 \left\{ S^\dagger, \frac{\boldsymbol{\sigma} \cdot e\mathbf{B}^{em}}{2m} \right\} S$$

$$V_1 = \Big(\text{hard}\Big) \times \Big(\text{soft}\Big)$$

- $\Big(\text{hard}\Big) = c_F^{\text{em}} = 1 + \frac{2\alpha_{\text{s}}(m_c)}{3\pi} + \cdots$
- Since $\boldsymbol{\sigma} \cdot e\mathbf{B}^{em}(\mathbf{R})$ behaves like the identity operator to all orders V_1 does not get soft contributions.



Diagrammatic factorization of the magnetic dipole coupling in the SU(3)$_f$ limit.

- *The argument is similar to the factorization of the QCD corrections in* $b \to u\, e^-\, \bar{\nu}_e$*, which leads to* $\mathcal{L}_{\rm eff} = -4G_F/\sqrt{2}\; V_{ub}\; \bar{e}_L \gamma_\mu \nu_L\; \bar{u}_L \gamma^\mu b_L$ *to all orders in* $\alpha_{\rm s}$*.*

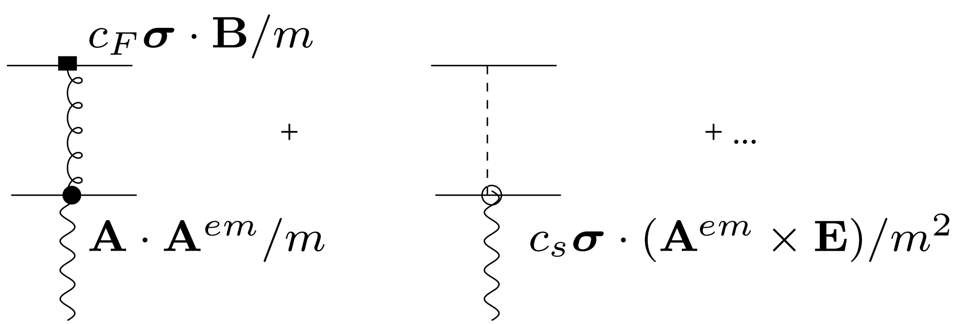
M1 operator at $\mathcal{O}(1)$

$$V_1 \left\{ S^\dagger, \frac{\boldsymbol{\sigma} \cdot e\mathbf{B}^{em}}{2m} \right\} S$$

- $V_1 = 1 + \dfrac{2\alpha_s(m_c)}{3\pi} + \cdots$
- No large quarkonium anomalous magnetic moment!
  (see also the lattice calculation of Dudek Edwards Richards 06)

M1 operators at $\mathcal{O}(v^2)$

$$\frac{1}{4m^2}\,\frac{V_2}{r}\,\left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times ee_Q\mathbf{B}^{\rm em}\right)\right]\right\}\mathrm{S}$$ and $$\frac{1}{4m^2}\,\frac{V_3}{r}\,\left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot ee_Q\mathbf{B}^{\rm em}\right\}\mathrm{S}$$



= (hard) $\times$ (soft)

- to all orders (hard) $= 2c_F - c_s = 1$; (soft) $= r^2 V_s'/2$

  (due to reparametrization/Poincaré invariance) Brambilla Gromes Vairo 03

- Therefore $V_2 = r^2 V_s'/2$ and $V_3 = 0$

- No scalar interaction!

M1 operators at $\mathcal{O}(v^2)$

$$V_4 \left\{ S^\dagger, \frac{\boldsymbol{\sigma} \cdot e\mathbf{B}^{em}}{4m^3} \right\} \boldsymbol{\nabla}_r^2 S$$

$$V_4 = \Big(\text{hard}\Big) \times \Big(\text{soft}\Big)$$

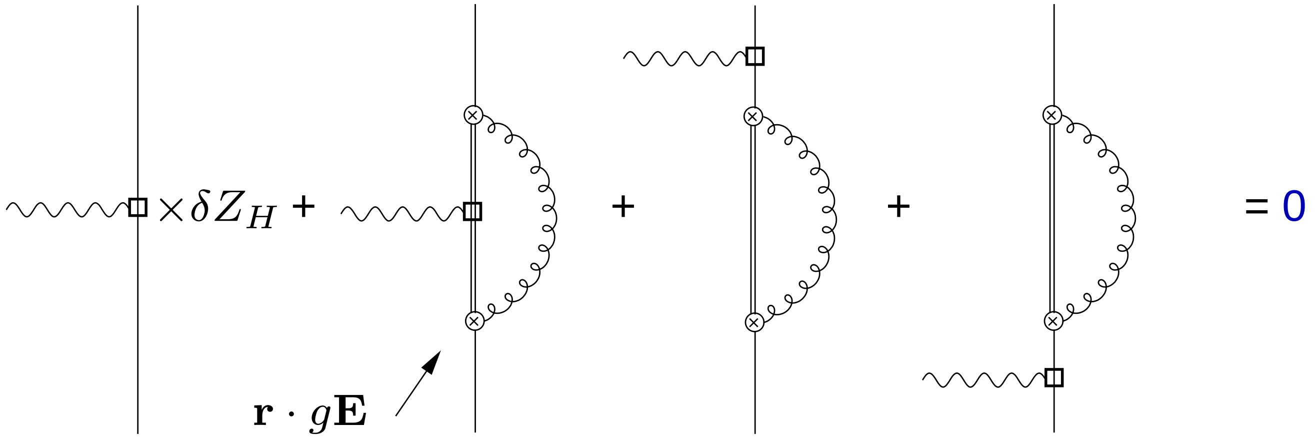
- $\Big(\text{hard}\Big) = 1$ *due to reparametrization invariance* Manohar 97
- $V_4 = 1 + \mathcal{O}(\alpha_s \text{ soft contributions})$

$$J/\psi \to \eta_c \gamma$$

$$\Gamma_{J/\psi \to \eta_c \gamma} = \int \frac{d^3k}{(2\pi)^3} (2\pi)\delta(E_p^{J/\psi} - k - E_k^{\eta_c}) |\langle \gamma(k)\eta_c | \mathcal{L}_\gamma | J/\psi \rangle|^2$$

$\mathcal{O}(v^2)$ corrections to the quarkonium states

Coupling of photons with octets: $V_1 \left\{ O^\dagger, \frac{\boldsymbol{\sigma} \cdot e\mathbf{B}^{em}}{2m} \right\} O$ (if $mv \gg \Lambda_{\rm QCD}$)



- If $mv^2 \sim \Lambda_{\rm QCD}$ the above graphs are potentially of order $\Lambda_{\rm QCD}^2/(mv)^2 \sim v^2$.
- The contribution vanishes because $\boldsymbol{\sigma} \cdot e\mathbf{B}^{em}(\mathbf{R})$ behaves like the identity operator.

- There are no non-perturbative contributions at $\mathcal{O}(v^2)$!

$J/\psi \to \eta_c\gamma$

Up to order v^2 the transition $J/\psi \to \eta_c\gamma$ is completely accessible by perturbation theory.

$$\Gamma(J/\psi \to \eta_c\gamma) = \frac{16}{3}\alpha e_c^2 \frac{k_\gamma^3}{M_{J/\psi}^2}\left[1 + C_F\frac{\alpha_s(M_{J/\psi}/2)}{\pi} - \frac{2}{3}(C_F\alpha_s(p_{J/\psi}))^2\right]$$

The normalization scale for the α_s inherited from κ_c is the charm mass ($\alpha_s(M_{J/\psi}/2) \approx 0.35 \sim v^2$), and for the α_s, which comes from the Coulomb potential, is the typical momentum transfer $p_{J/\psi} \approx mC_F\alpha_s(p_{J/\psi})/2 \approx 0.8$ GeV $\sim mv$.

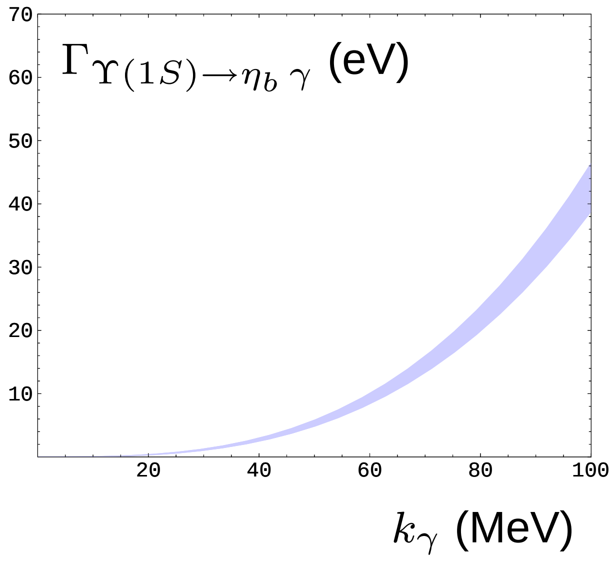
$\Gamma(J/\psi \to \eta_c\gamma) = (1.5 \pm 1.0)$ keV.

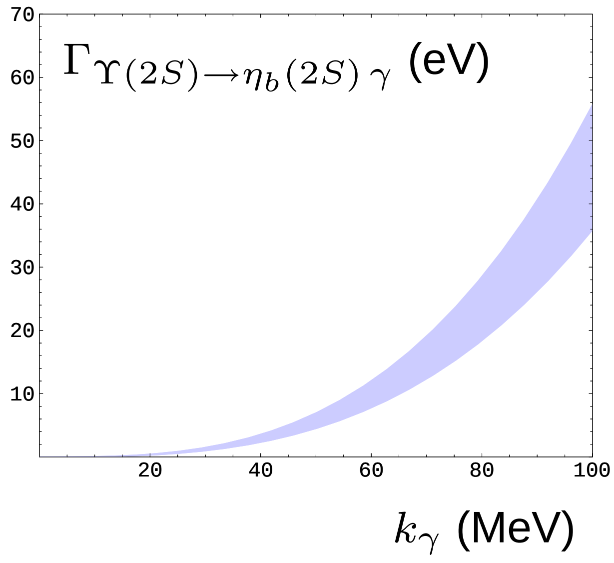
$$\Gamma_{J/\psi\to\gamma\eta_c} = \frac{16}{3}\alpha e_c^2 \frac{k_\gamma^3}{M_{J/\Psi}^2}\left(1 + \frac{4}{3}\frac{\alpha_s(M_{J/\Psi}/2)}{\pi} - \frac{2}{3}\frac{\langle 1|rV_s'|1\rangle}{M_{J/\Psi}} + 2\frac{\langle 1|V_s|1\rangle}{M_{J/\Psi}}\right)$$

- If $V_s = -\frac{4}{3}\frac{\alpha_s(\mu)}{r}$: $-\frac{2}{3}\frac{\langle 1|rV_s'|1\rangle}{M_{J/\Psi}} + 2\frac{\langle 1|V_s|1\rangle}{M_{J/\Psi}} = -\frac{32}{27}\alpha_s(\mu)^2 < 0$
- *If* $V_s = \sigma r$*:* $-\frac{2}{3}\frac{\langle 1|rV_s'|1\rangle}{M_{J/\Psi}} + 2\frac{\langle 1|V_s|1\rangle}{M_{J/\Psi}} = \frac{4}{3}\frac{\sigma}{M_{J/\Psi}}\langle 1|r|1\rangle > 0$

  A scalar interaction would add a negative contribution $-2\langle 1|V^{\text{scalar}}|1\rangle/M_{J/\Psi}$.

$\Gamma_{\Upsilon(1S)\to\eta_b\gamma}$ and $\Gamma_{\Upsilon(2S)\to\eta_b(2S)\,\gamma}$





$$\mathcal{B}_{\Upsilon(1S)\to\eta_b\gamma} = (6.8 \pm 5.5) \times 10^{-5}$$

M1 hindered transitions

- Two new operators contribute:

$$-\frac{1}{16m^2}\, c_S^{\rm em}\, \left[{\rm S}^\dagger, \boldsymbol{\sigma}\cdot\left[-i\boldsymbol{\nabla}\times, ee_Q\mathbf{E}^{\rm em}\right]\right]{\rm S}$$

and

$$-\frac{1}{16m^2}\, c_S^{\rm em}\, \left[{\rm S}^\dagger, \boldsymbol{\sigma}\cdot\left[-i\boldsymbol{\nabla}_r\times, \mathbf{r}^i(\boldsymbol{\nabla}^i ee_Q\mathbf{E}^{\rm em})\right]\right]{\rm S}$$

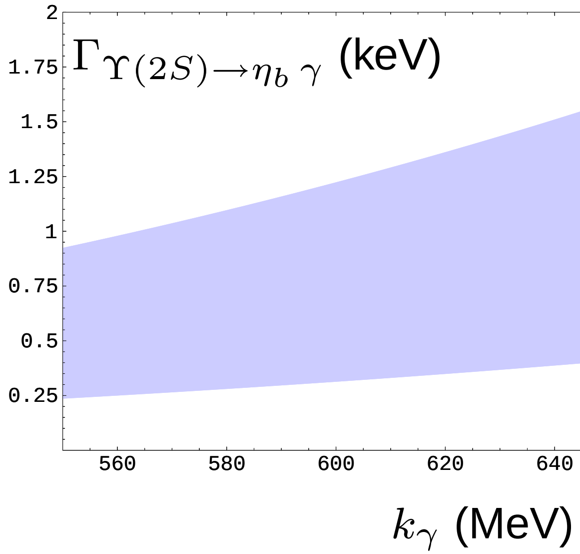
- Two new wave function corrections contribute:

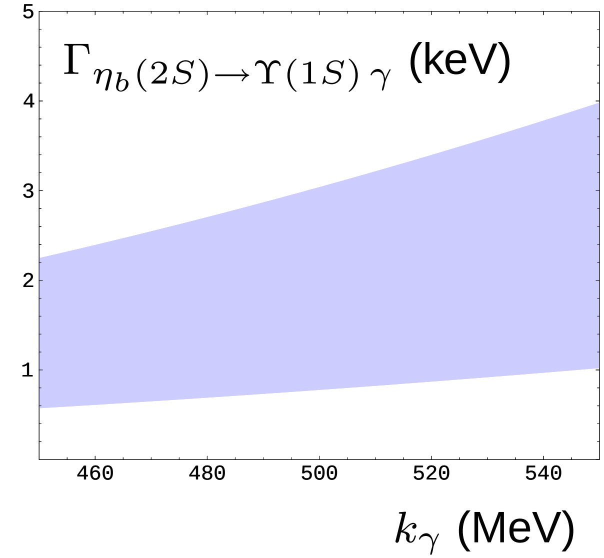
(1) induced by the spin-spin potential;

(2) recoil correction induced by the spin-orbit potential;

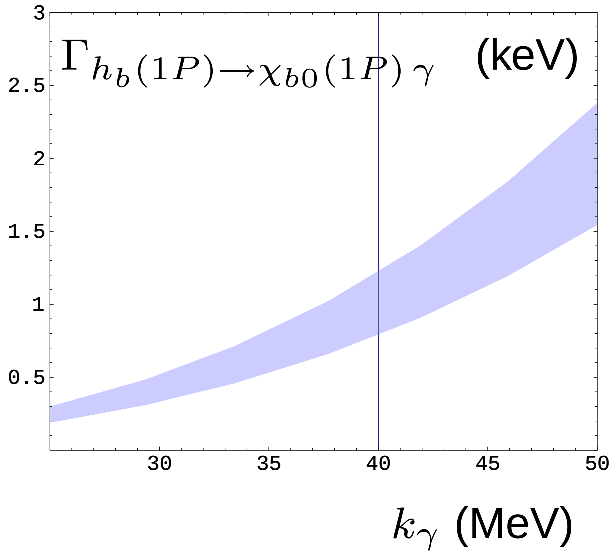
Due to the recoil, the final state develops a nonzero P-wave component suppressed by a factor $v\,k_\gamma/m$, which, in a $n^3S_1 \longrightarrow n'\,^1S_0\,\gamma$ transition, can be reached from the initial $^3S_1$ state through a $1/v$ enhanced E1 transition.

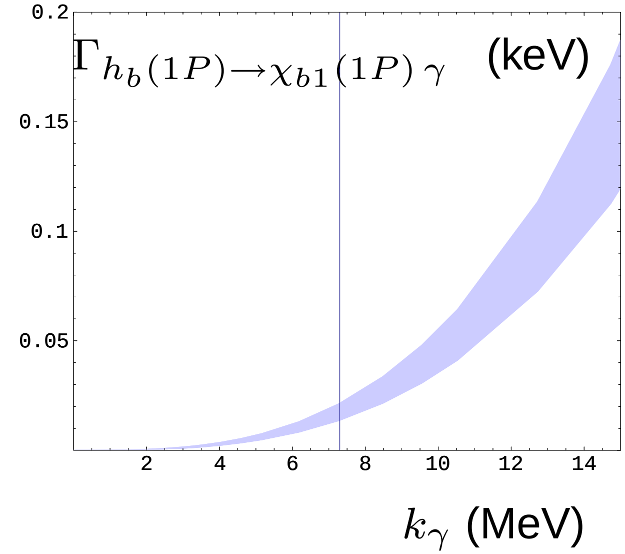
$\Gamma_{\Upsilon(2S)\to\eta_b\gamma}$ and $\Gamma_{\eta_b(2S)\to\Upsilon(1S)\,\gamma}$

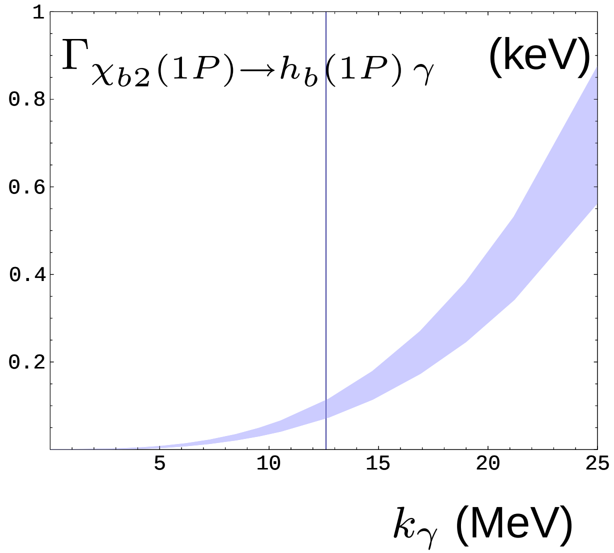




$\Gamma_{h_b(1P)\to\chi_{b0}(1P)\,\gamma}$, $\Gamma_{h_b(1P)\to\chi_{b1}(1P)\,\gamma}$ and $\Gamma_{\chi_{b2}(1P)\to h_b(1P)\,\gamma}$







$\Gamma_{h_b(1P)\to\chi_{b0}(1P)\,\gamma} = 1 \pm 0.2$ keV

$\Gamma_{h_b(1P)\to\chi_{b1}(1P)\,\gamma} = 17 \pm 4$ eV

$\Gamma_{\chi_{b2}(1P)\to h_b(1P)\,\gamma} = 90 \pm 20$ eV

Conclusions

The Grotch Owen Sebastian 84 formula holds under the following conditions:

- There is no scalar interaction.
- The quarkonium anomalous magnetic moment is small and positive: $2\alpha_s/(3\pi) + \ldots$.
- The formula is valid only in the weak coupling regime (i.e. for the lowest quarkonium resonances).
- It is valid up to relative order α_s^2.

* In the strong coupling regime (which applies to most of the charmonium resonances) at relative order v^2 much more terms than those predicted by naive potential models appear.
The EFT allows to express them as Wilson loop amplitudes to be calculated eventually on the lattice.