EFTs for Baryons with Two and Three Heavy Quarks

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based on

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Effective Field Theory Lagrangians for Baryons with Two and Three Heavy Quarks

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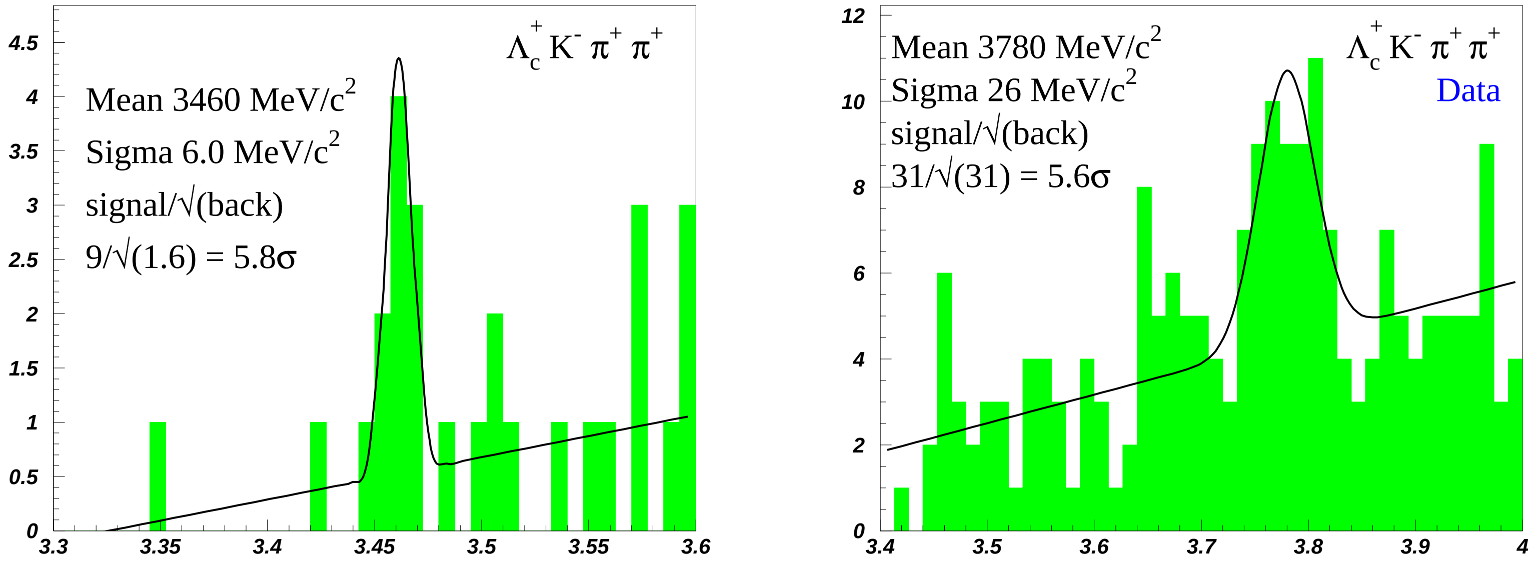
Experimental Invitation

Ξ_{cc} Doubly Charmed Baryons

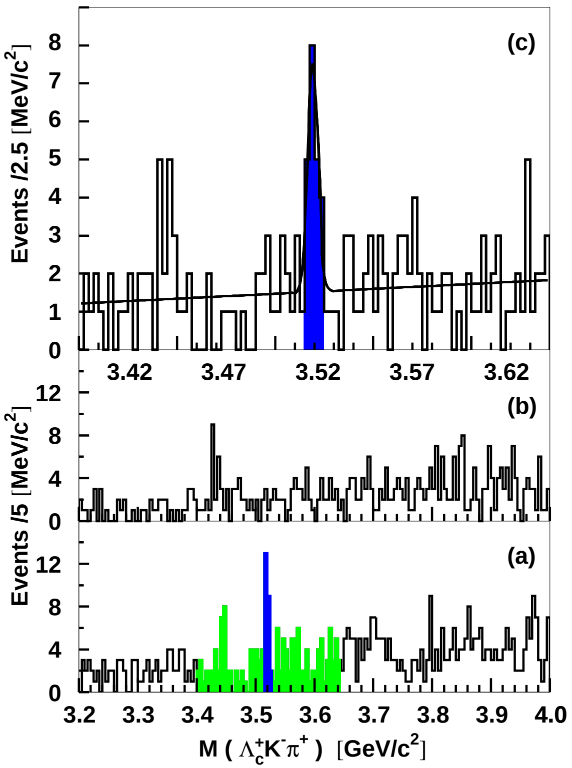
Evidence of 5 doubly charmed baryons ($ccd^+(3443)$, $ccd^+(3520)$, $ccu^{++}(3460)$, $ccu^{++}(3541)$, $ccu^{++}(3780)$) so far.



SELEX 02 04

Confirmation of $\Xi_{cc}^{+}(ccd)$ in two decay modes at a mass of $3518.7 \pm 1.7 \mathrm{MeV}, \tau < 33 fs$

- $\Xi_{cc}^{+} \to \Lambda_c^{+} K^{-} \pi^{+}$



22 events in signal region, expected background 6.1

- $\Xi_{cc}^{+} \to p D^{+} K^{-}$

5.4 events in signal region, expected background 1.6

Theoretical Invitation

HQET for Doubly Charmed Baryons

- In the case of $\bar{Q}q$ mesons the lowest lying $S = 0, 1$ states are degenerate in the $m \to \infty$ limit.
- The interaction Lagrangian that gives rise to the mass splitting at $\mathcal{O}(\Lambda_{\rm QCD}/m)$ is:

$$\frac{1}{m}\, Q^\dagger\, \frac{\boldsymbol{\sigma}}{2} \cdot g\mathbf{B}\, Q$$

HQET for Doubly Charmed Baryons

- In the case of QQq baryons, if the QQ are so close that they may be treated as a point-like spin 1 color antitriplet, then the system behaves like a $\bar{Q}q$ meson.
- The lowest lying $S = 1/2,\ 3/2$ states are degenerate in the $m \to \infty$ limit.
- The interaction Lagrangian that gives rise to the mass splitting at $\mathcal{O}(\Lambda_{\mathrm{QCD}}/m)$ is:

$$\frac{1}{m}\, V_i^\dagger\, i\epsilon_{ijk}\, g\mathbf{B}_k\, V_j$$

Savage Wise 90

HQET for Doubly Charmed Baryons

From the HQET symmetries:

$$M_{\Xi^*_{cc}} - M_{\Xi_{cc}} = \frac{3}{2}(M_{D^*} - M_D)$$

Savage Wise 90

$M_{\Xi^*_{cc}} - M_{\Xi_{cc}} \approx 240$ MeV

however

- potential models typically predict $\approx$ 60-120 MeV.
- lattice calculations predict $\approx$ 80-90 MeV.

Summary

1. Introduction to EFTs for heavy quarkonium

2. EFTs for QQq baryons

2.1 Ground state hyperfine splittings

3. EFTs for QQQ baryons

3.1 Weakly coupled bound state

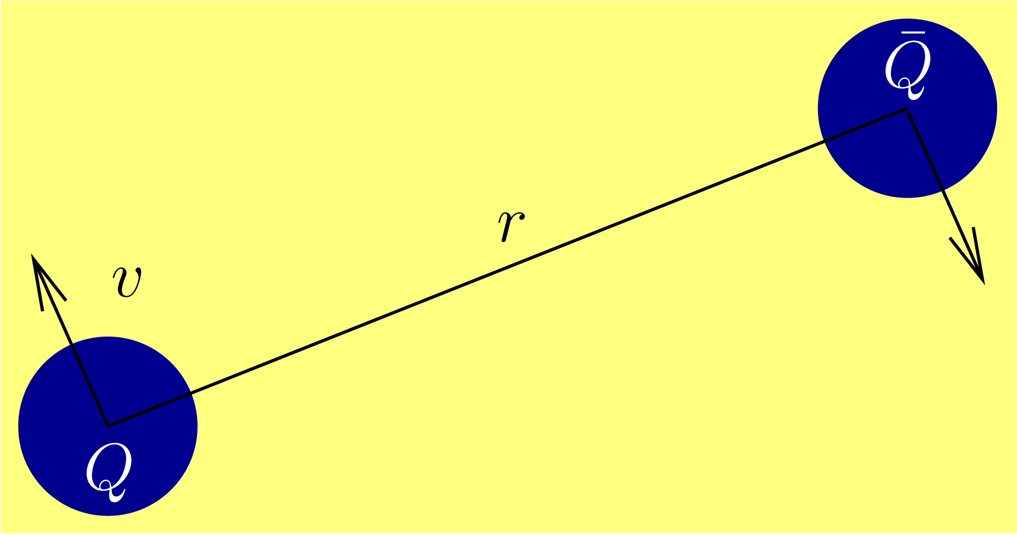
3.2 Strongly coupled bound state

3.3 Lattice results

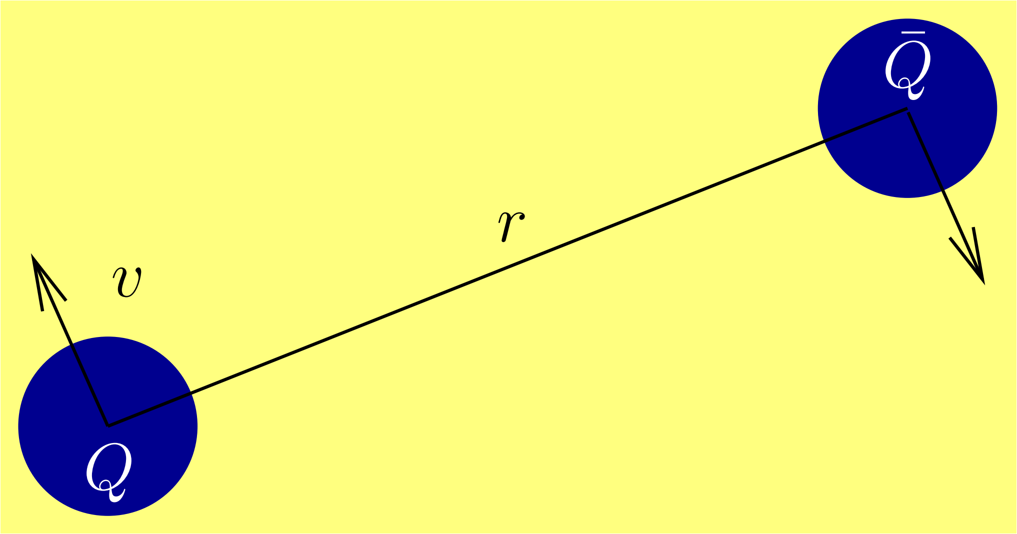
4. Outlook

1. EFTs for $Q\bar{Q}$

Quarkonium Scales



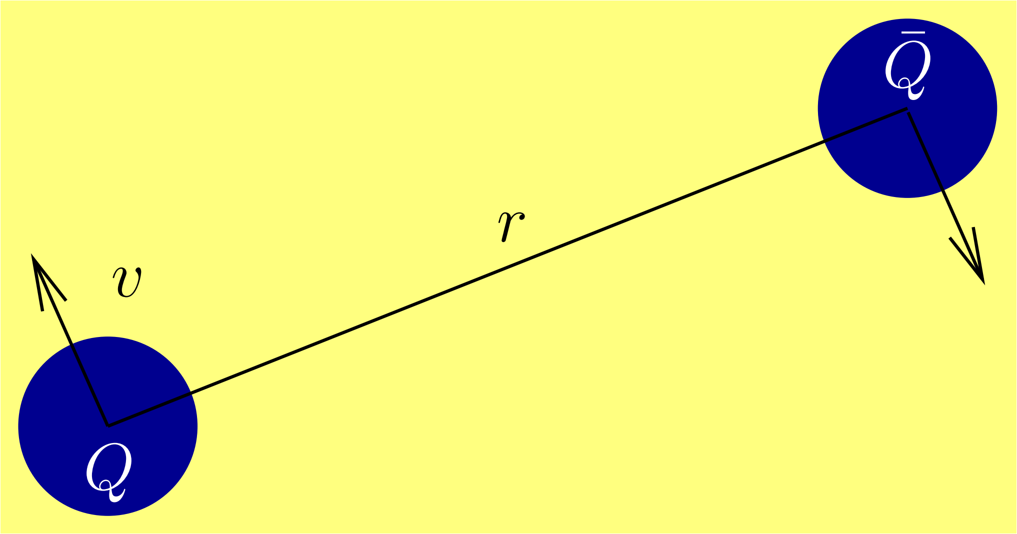
Quarkonium Scales



The mass scale is perturbative:

$m \gg \Lambda_{\mathrm{QCD}}$

Quarkonium Scales



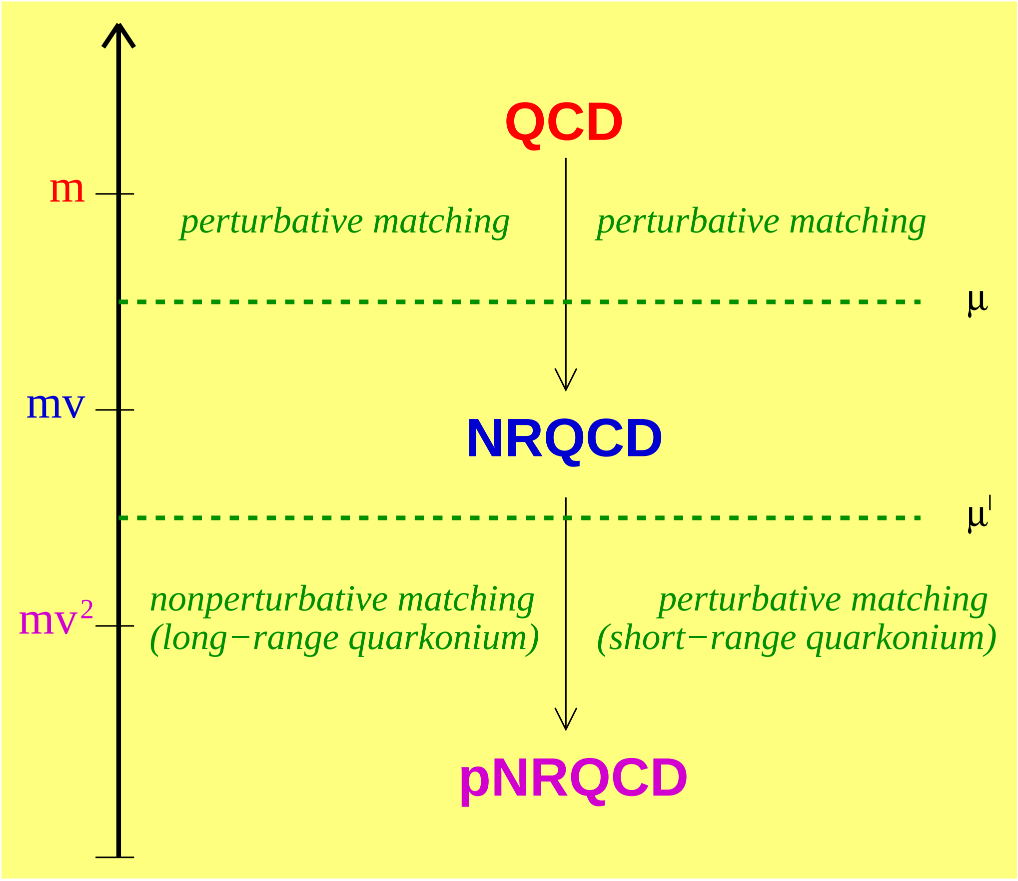
The mass scale is perturbative:

$m \gg \Lambda_{\rm QCD}$

The system is non-relativistic:

$m \gg 1/r \sim mv \gg E \sim mv^2$

Quarkonium Scales



A way to disentangle rigorously these scales is by substituting QCD scale by scale with simpler but equivalent Effective Field Theories.

Effective Field Theories

Let H be a system described by a fundamental Lagrangian $\mathcal{L}$.
Suppose H characterized by 2 scales: $\Lambda \gg \lambda$.

The EFT Lagrangian, $\mathcal{L}_{\rm EFT}$, suitable to describe H at scales lower than Λ is defined by
(1) a cut off $\Lambda \gg \mu \gg \lambda$;
(2) by some degrees of freedom that exist at scales lower than μ

$\Rightarrow$ $\mathcal{L}_{\rm EFT}$ is made of all operators O_n that may be built from the effective degrees of freedom and are consistent with the symmetries of $\mathcal{L}$.

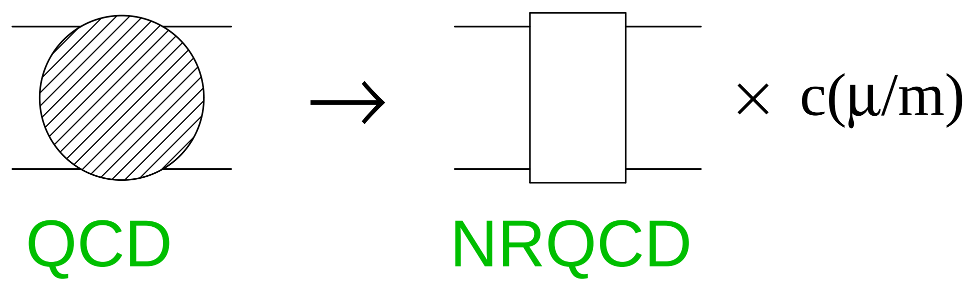
Effective Field Theories

$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c(\Lambda/\mu)\frac{O_n(\mu,\lambda)}{\Lambda^n}$$

- Since $\langle O_n \rangle \sim \lambda^n$ the EFT is organized as an expansion in λ/Λ.
- The EFT is renormalizable order by order in λ/Λ.
- The matching coefficients $c(\Lambda/\mu)$ encode the non-analytic behaviour in Λ. They are calculated by imposing that $\mathcal{L}_{\mathrm{EFT}}$ and $\mathcal{L}$ describe the same physics at any finite order in the expansion: matching procedure.
- If $\Lambda \gg \Lambda_{\mathrm{QCD}}$ then $c(\Lambda/\mu)$ may be calculated in perturbation theory.

NRQCD

NRQCD is the EFT that follows from QCD when $\Lambda = m$



NRQCD

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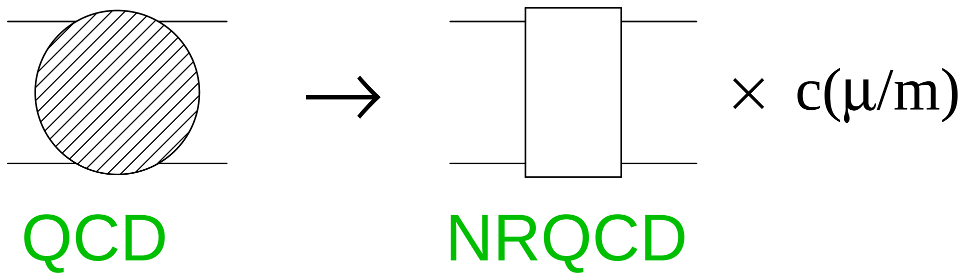


- The matching is perturbative.
- The Lagrangian is organized as an expansion in v and $\alpha_s(m)$: $\mathcal{L}_{\mathrm{NRQCD}} = \sum_n c(\alpha_s(m/\mu)) \times O_n(\mu, \lambda)/m^n$.

Caswell Lepage 86, Bodwin Braaten Lepage 95

NRQCD

$$\mathcal{L} = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + c_F \frac{\mathbf{S}\cdot g\mathbf{B}}{m} + c_D \frac{[\mathbf{D}\cdot, g\mathbf{E}]}{8m^2} + \dots \right) \psi$$

$$1 + (\cdots)\, \alpha_{\mathrm{s}} + \cdots$$

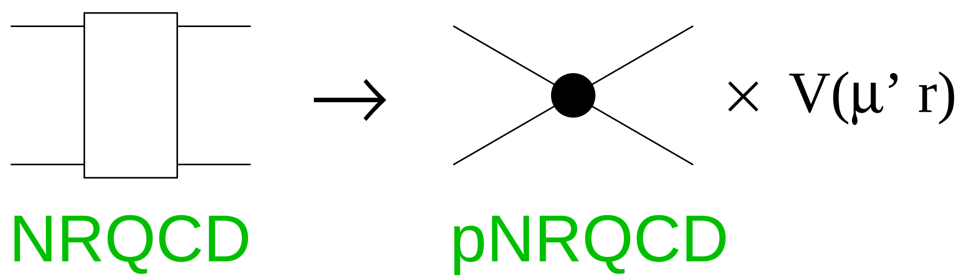
$$f = \mathrm{Re} f + i \mathrm{Im} f$$

$$+ \sum_K \frac{f}{m^2} \psi^\dagger \, K \, \chi \chi^\dagger \, K \, \psi + \cdots$$

$$- \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum^{n_f} \bar{q}\, i \not{D}\, q + \dots$$

pNRQCD ($mv \gg \Lambda_{\mathrm{QCD}}$)

pNRQCD is the EFT for heavy quarkonium that follows from NRQCD when $\Lambda = 1/r \sim mv$



pNRQCD ($mv \gg \Lambda_{\mathrm{QCD}}$)

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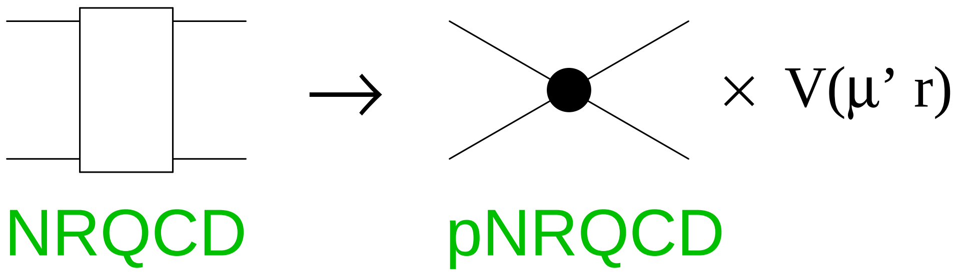
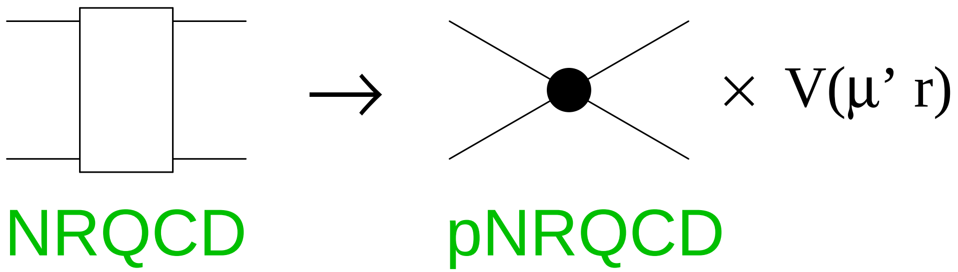


- If $mv \gg \Lambda_{\mathrm{QCD}}$, the matching is perturbative

pNRQCD ($mv \gg \Lambda_{\rm QCD}$)

pNRQCD is the EFT for heavy quarkonium that follows from NRQCD when $\Lambda = 1/r \sim mv$



- Degrees of freedom: quarks and gluons

Q-$\bar{Q}$ states, with energy $\sim \Lambda_{\rm QCD}, mv^2$

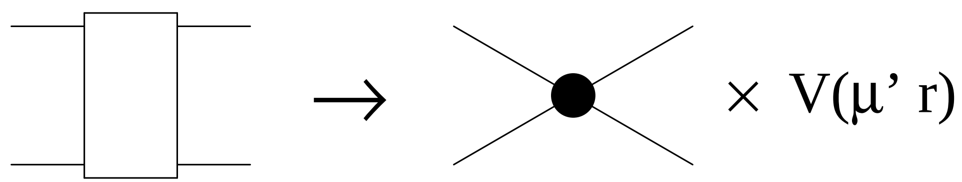
momentum $\lesssim mv$

$\Rightarrow$ (i) singlet S (ii) octet O

Gluons with energy and momentum $\sim \Lambda_{\rm QCD}, mv^2$

pNRQCD ($mv \gg \Lambda_{\rm QCD}$)

pNRQCD is the EFT for heavy quarkonium that follows from NRQCD when $\Lambda = 1/r \sim mv$



NRQCD pNRQCD

- $$\mathcal{L}_{\rm pNRQCD} = \sum_n c(\alpha_{\rm s}(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

Non-analytic behaviour in $r \to$ matching coefficients V

pNRQCD ($mv \gg \Lambda_{\mathrm{QCD}}$)

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \mathrm{Tr}\left\{\mathrm{S}^\dagger\left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s\right)\mathrm{S} + \mathrm{O}^\dagger\left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o\right)\mathrm{O}\right\}$$

LO in r

$$+V_A \mathrm{Tr}\left\{\mathrm{O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{S} + \mathrm{S}^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{O}\right\}$$
$$+\frac{V_B}{2}\mathrm{Tr}\left\{\mathrm{O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{O} + \mathrm{O}^\dagger\mathrm{O}\mathbf{r}\cdot g\mathbf{E}\right\}$$

NLO in r

Pineda Soto 97, Brambilla Pineda Soto Vairo 99, 00, 03

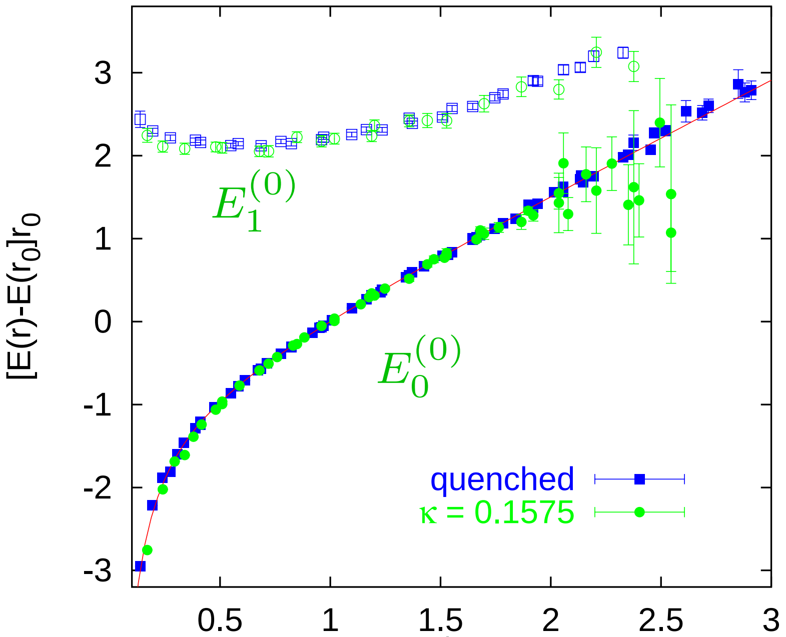
pNRQCD ($mv \sim \Lambda_{\mathrm{QCD}}$)

- All quarks with energy $\gg mv^2$ and momentum $\gg mv$ are integrated out.

pNRQCD ($mv \sim \Lambda_{\rm QCD}$)

- All quarks with energy $\gg mv^2$ and momentum $\gg mv$ are integrated out.

- All gluonic excitations between heavy quarks are integrated out since they develop a gap of order $\Lambda_{\rm QCD}$ with the static $Q\bar{Q}$ energy.



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($r_0 \simeq 0.5$ fm)

pNRQCD ($mv \sim \Lambda_{\rm QCD}$)

- All quarks with energy $\gg mv^2$ and momentum $\gg mv$ are integrated out.

- All gluonic excitations between heavy quarks are integrated out since they develop a gap of order $\Lambda_{\rm QCD}$ with the static $Q\bar{Q}$ energy.

$\Rightarrow$ The singlet quarkonium field S of energy mv^2 and momentum mv is the only the degree of freedom of pNRQCD(up to ultrasoft light quarks, e.g. pions).

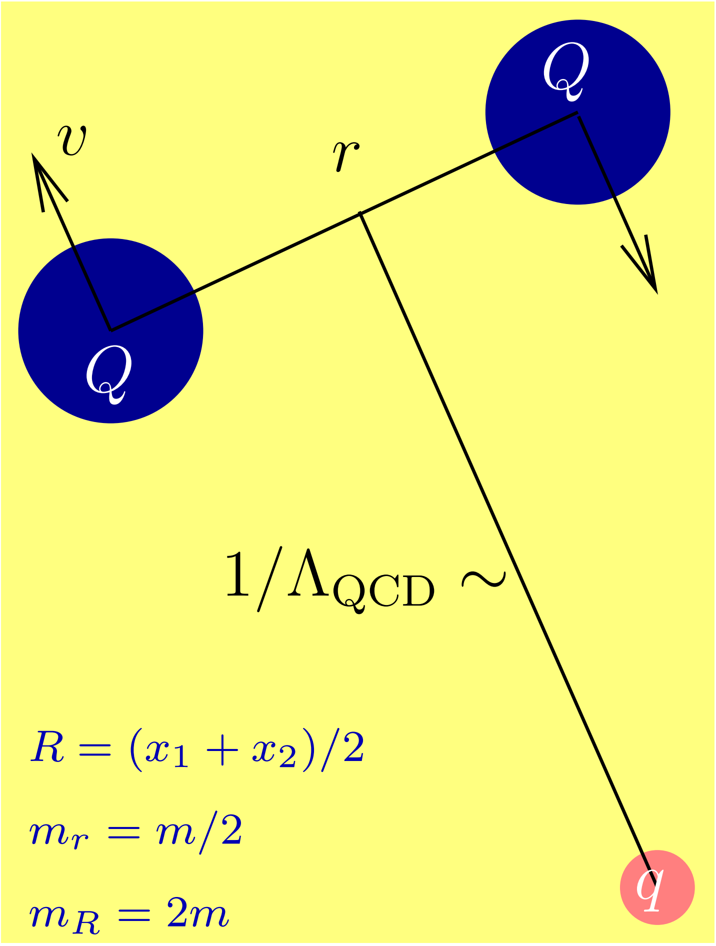
pNRQCD ($mv \sim \Lambda_{\rm QCD}$)

$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\}$$

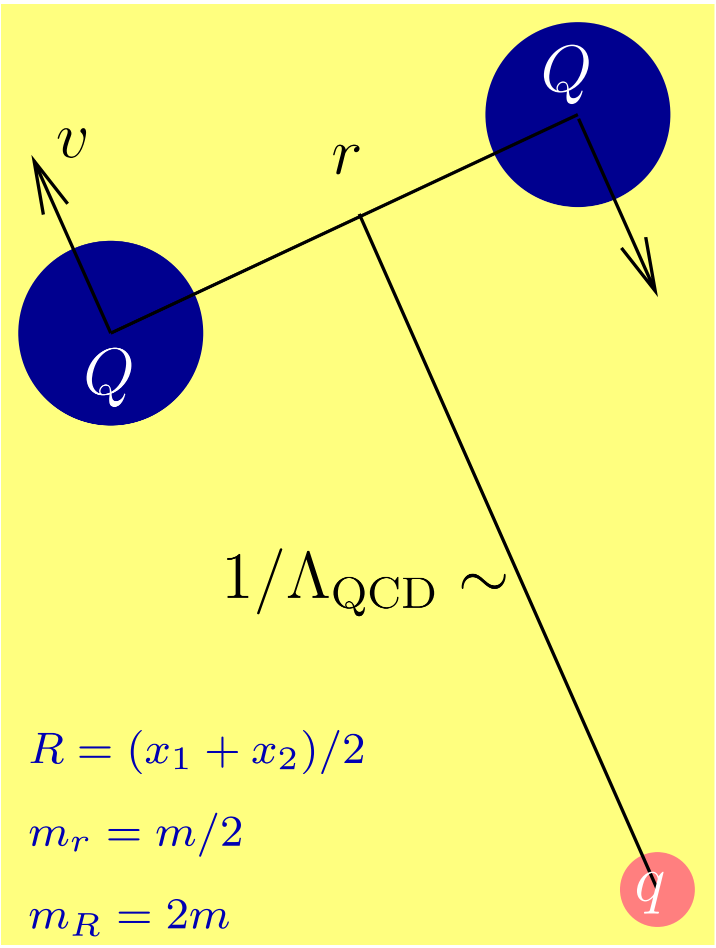
- *The potential V_s ($\mathrm{Re}\, V_s + i\, \mathrm{Im}\, V_s$) is a mixture of perturbative and non-perturbative contributions to be determined by the matching.*

2. EFTs for QQq

QQq Scales



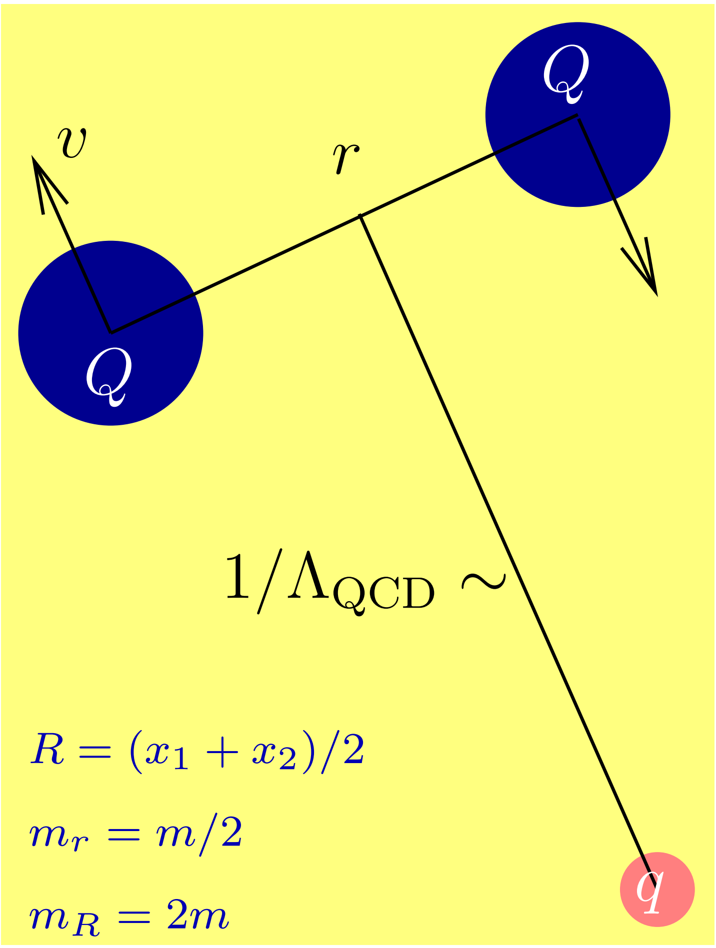
QQq Scales



The mass scale is perturbative:

$m \gg \Lambda_{\mathrm{QCD}}$

QQq Scales



The mass scale is perturbative:

$m \gg \Lambda_{\rm QCD}$

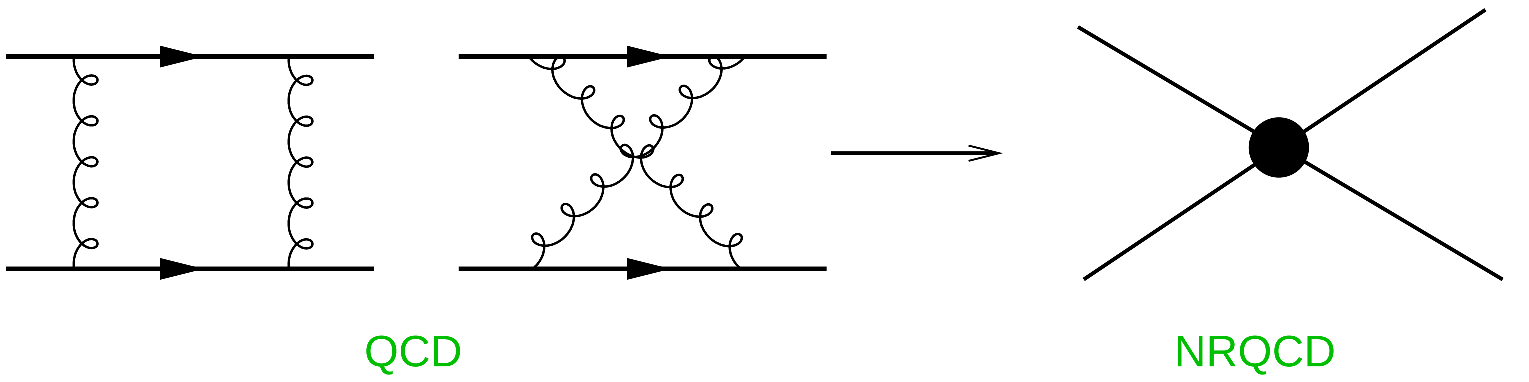
The system is characterized by the scales:

$m \gg 1/r \sim mv \gg E \sim mv^2$

and $\Lambda_{\rm QCD}$.

We will assume $mv \gg \Lambda_{\rm QCD}$.

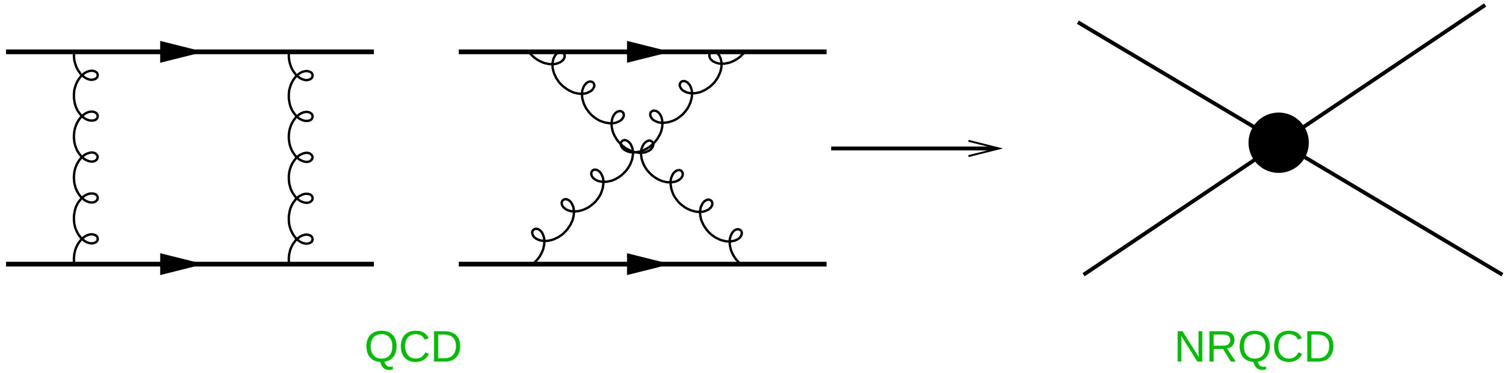
NRQCD



$$\mathcal{L}_{QQ}^{\mathrm{NRQCD}} = \frac{d_{QQ}^{ss}}{m^2} Q^\dagger Q Q^\dagger Q + \frac{d_{QQ}^{sv}}{m^2} Q^\dagger \boldsymbol{\sigma} Q \cdot Q^\dagger \boldsymbol{\sigma} Q$$

$$+ \frac{d_{QQ}^{vs}}{m^2} \sum_{a=1}^{8} Q^\dagger \mathrm{T}^a Q Q^\dagger \mathrm{T}^a Q + \frac{d_{QQ}^{vv}}{m^2} \sum_{a=1}^{8} Q^\dagger \mathrm{T}^a \boldsymbol{\sigma} Q \cdot Q^\dagger \mathrm{T}^a \boldsymbol{\sigma} Q$$

NRQCD



$$d_{QQ}^{ss} = C_F \left(\frac{C_A}{2} - C_F\right) \alpha_s^2 \left(\ln \frac{m^2}{\mu^2} - \frac{2}{3}\right)$$

$$d_{QQ}^{sv} = C_F \left(\frac{C_A}{2} - C_F\right) \alpha_s^2$$

$$d_{QQ}^{vs} = 2C_F \alpha_s^2 \left(\ln \frac{m^2}{\mu^2} - \frac{2}{3}\right) - \frac{1}{4} C_A \alpha_s^2 \left(\ln \frac{m^2}{\mu^2} - \frac{23}{3}\right)$$

$$d_{QQ}^{vv} = 2C_F \alpha_s^2 - \frac{C_A \alpha_s^2}{4} \left(\ln \frac{m^2}{\mu^2} + 7\right)$$

pNRQCD ($mv \gg \Lambda_{\rm QCD}$)

- *The (weakly coupled) EFT for* QQq *baryons contains:*
  $(QQ)_{\bar{3}} = (T^1, T^2, T^3)$, $(QQ)_6 = (\Sigma^1, ..., \Sigma^6)$, q *and gluons.*

$$Q_{1\,i}(\boldsymbol{x}_1)Q_{2\,j}(\boldsymbol{x}_2) \sim \sum_{\ell=1}^{3} T^\ell(\boldsymbol{r},\boldsymbol{R})\,\underline{\mathbf{T}}^\ell_{ij} + \sum_{\sigma=1}^{6} \Sigma^\sigma(\boldsymbol{r},\boldsymbol{R})\,\underline{\boldsymbol{\Sigma}}^\sigma_{ij} \quad i,j = 1,2,3$$

$$\underline{\mathbf{T}}^\ell_{ij} = \frac{1}{\sqrt{2}}\epsilon_{\ell ij},$$

$$\underline{\boldsymbol{\Sigma}}^1_{11} = \underline{\boldsymbol{\Sigma}}^4_{22} = \underline{\boldsymbol{\Sigma}}^6_{33} = 1,$$

$$\underline{\boldsymbol{\Sigma}}^2_{12} = \underline{\boldsymbol{\Sigma}}^2_{21} = \underline{\boldsymbol{\Sigma}}^3_{13} = \underline{\boldsymbol{\Sigma}}^3_{31} = \underline{\boldsymbol{\Sigma}}^5_{23} = \underline{\boldsymbol{\Sigma}}^5_{32} = \frac{1}{\sqrt{2}},$$

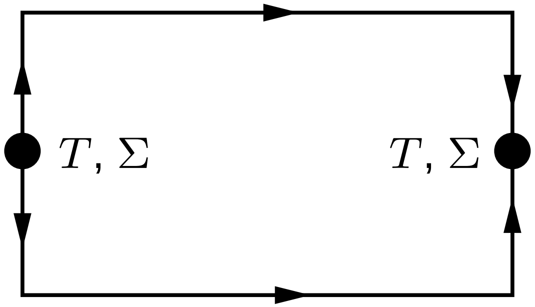
all other entries are zero.

pNRQCD ($mv \gg \Lambda_{\mathrm{QCD}}$)

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_{f=1}^{3} \bar{q}_f \, i\not{D} q_f$$
$$+\delta\mathcal{L}^{(0,0)}_{\text{pNRQCD}} + \delta\mathcal{L}^{(0,1)}_{\text{pNRQCD}} + \delta\mathcal{L}^{(1,0)}_{\text{pNRQCD}} + \cdots$$

pNRQCD ($mv \gg \Lambda_{\rm QCD}$)

$$\delta\mathcal{L}^{(0,0)}_{\text{pNRQCD}} = \int d^3r\, T^\dagger \left[iD_0 - V_T^{(0)}\right] T + \Sigma^\dagger \left[iD_0 - V_\Sigma^{(0)}\right] \Sigma$$



$$V_T^{(0)}(r) = -\frac{2}{3}\frac{\alpha_{\rm s}}{|\boldsymbol{r}|}$$

$$V_\Sigma^{(0)}(r) = \frac{1}{3}\frac{\alpha_{\rm s}}{|\boldsymbol{r}|}$$

pNRQCD ($mv \gg \Lambda_{\mathrm{QCD}}$)

$$\delta\mathcal{L}^{(0,1)}_{\text{pNRQCD}} = -\int d^3r\, V^{(0,1)}_{T\boldsymbol{r}\cdot\boldsymbol{E}\Sigma} \left[\left(\sum_{ijk=1}^{3} \underline{\mathbf{T}}^{\ell}_{ij} T^a_{jk} \underline{\mathbf{\Sigma}}^{\sigma}_{ki} \right) T^{\ell\dagger} \boldsymbol{r}\cdot g\boldsymbol{E}^a\, \Sigma^{\sigma} - \left(\sum_{ijk=1}^{3} \underline{\mathbf{\Sigma}}^{\sigma}_{ij} T^a_{jk} \underline{\mathbf{T}}^{\ell}_{ki} \right) \Sigma^{\sigma\dagger} \boldsymbol{r}\cdot g\boldsymbol{E}^a\, T^{\ell} \right]$$

pNRQCD ($mv \gg \Lambda_{\rm QCD}$)

$$\delta\mathcal{L}^{(1,0)}_{\text{pNRQCD}} = \int d^3r\, T^\dagger \left[\frac{\boldsymbol{D}_R^2}{4m} + \frac{\boldsymbol{\nabla}_r^2}{m}\right] T + \Sigma^\dagger \left[\frac{\boldsymbol{D}_R^2}{4m} + \frac{\boldsymbol{\nabla}_r^2}{m}\right] \Sigma$$

$$+V^{(1,0)}_{T\boldsymbol{\sigma}\cdot\boldsymbol{B}\Sigma} \left[\left(\sum_{ijk=1}^{3} \underline{\mathbf{T}}^{\ell}_{ij} T^a_{jk} \underline{\boldsymbol{\Sigma}}^{\sigma}_{ki}\right) T^{\ell\dagger} \frac{c_F}{2m} \left(-\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}\right) \cdot g\boldsymbol{B}^a\, \Sigma^\sigma\right.$$

$$\left. - \left(\sum_{ijk=1}^{3} \underline{\boldsymbol{\Sigma}}^{\sigma}_{ij} T^a_{jk} \underline{\mathbf{T}}^{\ell}_{ki}\right) \Sigma^{\sigma\dagger} \frac{c_F}{2m} \left(-\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}\right) \cdot g\boldsymbol{B}^a\, T^\ell\right]$$

$$+\frac{V^{(1,0)}_{T\boldsymbol{\sigma}\cdot\boldsymbol{B}T}}{2} T^\dagger \frac{c_F}{2m} \left(\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}\right) \cdot g\boldsymbol{B}^a T^a_{\bar{3}}\, T$$

$$+\frac{V^{(1,0)}_{\Sigma\boldsymbol{\sigma}\cdot\boldsymbol{B}\Sigma}}{2} \Sigma^\dagger \frac{c_F}{2m} \left(\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}\right) \cdot g\boldsymbol{B}^a T^a_{6}\, \Sigma + \cdots$$

pNRQCD ($mv \gg \Lambda_{\rm QCD}$)

$$\delta\mathcal{L}^{(1,0)}_{\text{pNRQCD}} = \int d^3r\, T^\dagger \left[\frac{\boldsymbol{D}_R^2}{4m} + \frac{\boldsymbol{\nabla}_r^2}{m}\right] T + \Sigma^\dagger \left[\frac{\boldsymbol{D}_R^2}{4m} + \frac{\boldsymbol{\nabla}_r^2}{m}\right] \Sigma$$

$$+V^{(1,0)}_{T\boldsymbol{\sigma}\cdot\boldsymbol{B}\Sigma} \left[\left(\sum_{ijk=1}^{3} \underline{\mathbf{T}}^{\ell}_{ij} T^a_{jk} \underline{\boldsymbol{\Sigma}}^{\sigma}_{ki} \right) T^{\ell\dagger} \frac{c_F}{2m} \left(-\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}\right) \cdot g\boldsymbol{B}^a\, \Sigma^\sigma \right.$$

$$\left. - \left(\sum_{ijk=1}^{3} \underline{\boldsymbol{\Sigma}}^{\sigma}_{ij} T^a_{jk} \underline{\mathbf{T}}^{\ell}_{ki} \right) \Sigma^{\sigma\dagger} \frac{c_F}{2m} \left(-\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}\right) \cdot g\boldsymbol{B}^a\, T^\ell \right]$$

$$+\frac{V^{(1,0)}_{T\boldsymbol{\sigma}\cdot\boldsymbol{B}T}}{2} T^\dagger \frac{c_F}{2m} \left(\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}\right) \cdot g\boldsymbol{B}^a T^a_{\bar{3}}\, T$$

$$+\frac{V^{(1,0)}_{\Sigma\boldsymbol{\sigma}\cdot\boldsymbol{B}\Sigma}}{2} \Sigma^\dagger \frac{c_F}{2m} \left(\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}\right) \cdot g\boldsymbol{B}^a T^a_{6}\, \Sigma + \cdots$$

Hyperfine Splittings: LO matching

- At the level of NRQCD:

$$\delta \mathcal{L}_{\text{NRQCD}} = Q_1^\dagger c_F \, \frac{\boldsymbol{\sigma} \cdot g\boldsymbol{B}}{2m} Q_1 + Q_2^\dagger c_F \, \frac{\boldsymbol{\sigma} \cdot g\boldsymbol{B}}{2m} Q_2 \,.$$

Hyperfine Splittings: LO matching

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$$\delta\mathcal{L}_{\rm NRQCD} = Q_1^\dagger c_F\, \frac{\boldsymbol{\sigma}\cdot g\boldsymbol{B}}{2m} Q_1 + Q_2^\dagger c_F\, \frac{\boldsymbol{\sigma}\cdot g\boldsymbol{B}}{2m} Q_2\,.$$

- Projecting onto $Q_{1\,i}\,Q_{2\,j}$ one obtains in the antitriplet-antitriplet sector

$$\begin{aligned}\delta\mathcal{L}_{\rm pNRQCD} &= \int d^3r \sum_{\ell\ell'\;ijk=1}^{3} \left(T^{\ell\dagger}\underline{\mathbf{T}}^{\ell}_{ij}\, \frac{c_F\boldsymbol{\sigma}^{(1)}}{2m}\cdot g\boldsymbol{B}^a T^a_{ik}\, T^{\ell'}\underline{\mathbf{T}}^{\ell'}_{kj} \right.\\ &\qquad \left. + T^{\ell\dagger}\underline{\mathbf{T}}^{\ell}_{ij}\, \frac{c_F\boldsymbol{\sigma}^{(2)}}{2m}\cdot g\boldsymbol{B}^a T^a_{jk}\, T^{\ell'}\underline{\mathbf{T}}^{\ell'}_{ik} \right)\end{aligned}$$

Hyperfine Splittings: LO matching

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- Projecting onto $Q_{1\,i}\,Q_{2\,j}$ one obtains in the antitriplet-antitriplet sector

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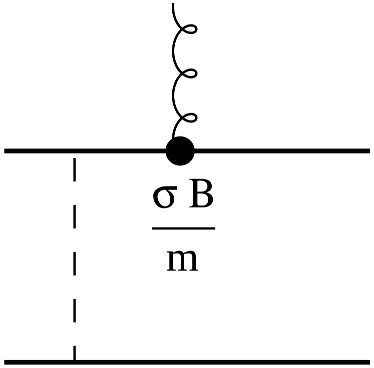
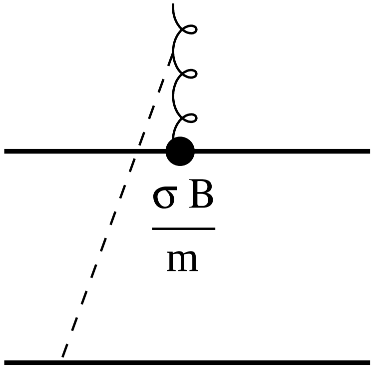
- Using $\sum_{ijk=1}^{3}\underline{\mathbf{T}}^{\ell}_{ij}T^a_{ik}\underline{\mathbf{T}}^{\ell'}_{kj} = \sum_{ijk=1}^{3}\underline{\mathbf{T}}^{\ell}_{ij}T^a_{jk}\underline{\mathbf{T}}^{\ell'}_{ik} = -T^a_{\ell'\ell}/2 = (T^a_{\bar{3}})_{\ell\ell'}/2$
one obtains

$$V^{(1,0)}_{T\boldsymbol{\sigma}\cdot\boldsymbol{B}T} = 1 \quad \text{at LO}$$

Hyperfine Splittings: NLO matching



cancel in the matching



$\propto$ external energies $\propto p^2/m \Rightarrow$ contribute to h.o. operators

$$V^{(1,0)}_{T\boldsymbol{\sigma}\cdot\boldsymbol{B}T} = 1 + \mathcal{O}(\alpha_s^2)$$

Hyperfine Splittings: the interaction

$$\frac{c_F(\mu)}{2m} \left(1+\mathcal{O}(\alpha_s^2)\right) \; T^\dagger \, \frac{\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}}{2} \, g\mathbf{B}^a T_{\bar{3}}^a \, T$$

- The triplet color normalization gives an extra $1/2$ factor with respect to the `Savage Wise 90` work.

Hyperfine Splittings Formula

Consider heavy-light mesons. We call P_Q and P_Q^* the $\bar{Q}u$ or $\bar{Q}d$ spin 0 or spin 1 states.

Consider the S-wave ground state of a doubly heavy baryon. Since an (anti)triplet state is antisymmetric in colour, due to the Fermi statistics, the two heavy quarks are allowed only in a spin 1 (symmetric) state. The lowest energy states for QQu or QQd are called Ξ_{QQ} (Ξ_{QQ}^*) for spin $1/2$ ($3/2$).

$$M_{\Xi_{QQ}^*} - M_{\Xi_{QQ}} = \frac{3\, m_{Q'}}{4\, m_Q} \frac{c_F^{(Q)}}{c_F^{(Q')}} \left(M_{P_{Q'}^*} - M_{P_{Q'}} \right) \left[1 + \mathcal{O}\left(\alpha_s^2, \frac{\Lambda_{\rm QCD}}{m_Q}, \frac{\Lambda_{\rm QCD}}{m_{Q'}} \right) \right]$$

Hyperfine Splittings Formula

$$M_{\Xi^*_{cc}} - M_{\Xi_{cc}} = 120 \pm 40 \ \ \text{MeV}$$

$$M_{\Xi^*_{bb}} - M_{\Xi_{bb}} = 34 \pm 4 \ \ \text{MeV}$$

Hyperfine Splittings Formula

$$M_{\Xi_{cc}^*} - M_{\Xi_{cc}} = 120 \pm 40 \ \ \mathrm{MeV}$$
$$M_{\Xi_{bb}^*} - M_{\Xi_{bb}} = 34 \pm 4 \ \ \mathrm{MeV}$$

To be compared with the lattice results:

$$M_{\Xi_{cc}^*} - M_{\Xi_{cc}} = 89 \pm 15 \ \ \mathrm{MeV}$$

Flynn Mescia Tariq 03 - quenched QCD

$$M_{\Xi_{cc}^*} - M_{\Xi_{cc}} = 80 \pm 10^{+3}_{-7} \ \ \mathrm{MeV}$$

Lewis Mathur Woloshyn 01 - quenched QCD

$$M_{\Xi_{bb}^*} - M_{\Xi_{bb}} = 20 \pm 6^{+2}_{-3} \ \ \mathrm{MeV}$$

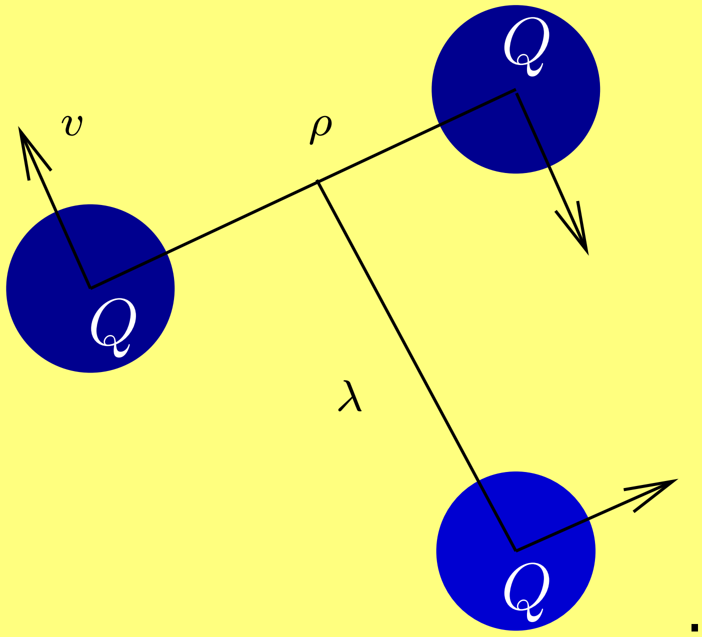
Ali Khan et al. 99 - quenched NRQCD

$$M_{\Xi_{bb}^*} - M_{\Xi_{bb}} = 20 \pm 6^{+3}_{-4} \ \ \mathrm{MeV}$$

Mathur Lewis Woloshyn 02 - quenched NRQCD

3. EFTs for QQQ

QQQ Scales



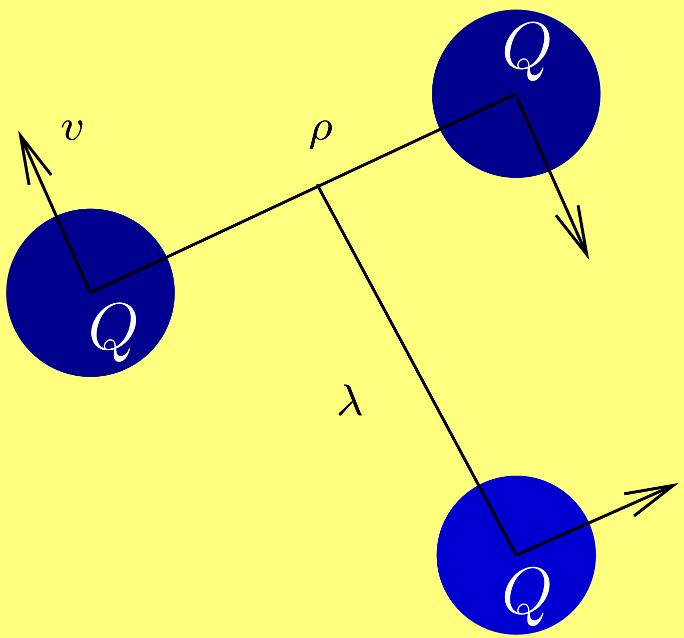
$R = (m(x_1 + x_2) + m_3 x_3)/m_R$

$m_\rho = m/2$

$m_\lambda = 2mm_3/m_R$

$m_R = 2m$

QQQ Scales



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The mass scale is perturbative:

$$m, m_3 \gg \Lambda_{\rm QCD}$$

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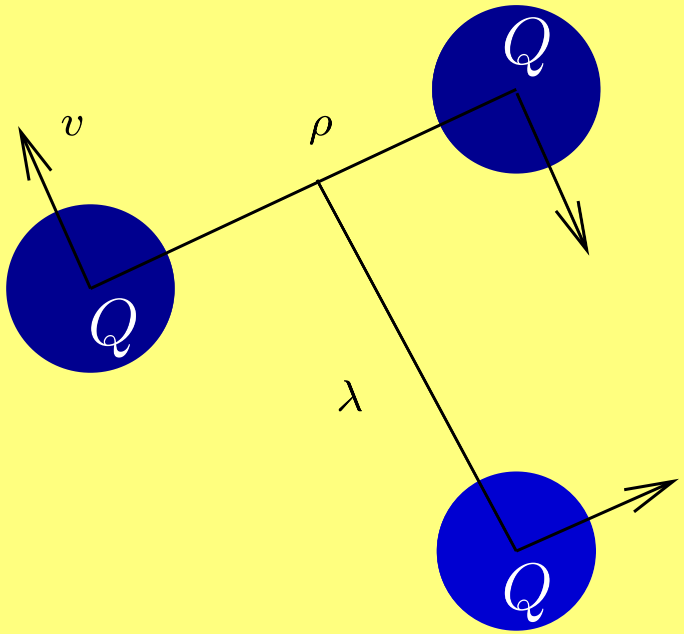


$R = (m(x_1 + x_2) + m_3 x_3)/m_R$

$m_\rho = m/2$

$m_\lambda = 2mm_3/m_R$

$m_R = 2m$

The mass scale is perturbative:

$m, m_3 \gg \Lambda_{\rm QCD}$

The system is characterized by the scales:

$m \gg 1/\rho \sim 1/\lambda \sim mv \gg E \sim mv^2$

and $\Lambda_{\rm QCD}$.

pNRQCD ($mv \gg \Lambda_{\rm QCD}$)

- *The (weakly coupled) EFT for* QQQ *baryons contains:* q, *gluons*, $(QQ)_1 = S$, $(QQ)_8 = (O^{\mathsf{A}\,1}, \dots, O^{\mathsf{A}\,8})$, $(QQ)_8 = (O^{\mathsf{S}\,1}, \dots, O^{\mathsf{S}\,8})$ *and* $(QQ)_{10} = (\Delta^1, \dots, \Delta^{10})$.

$$Q_{1\,i}(\boldsymbol{x}_1)Q_{2\,j}(\boldsymbol{x}_2)Q_{3\,k}(\boldsymbol{x}_3) \sim S(\boldsymbol{\rho}, \boldsymbol{\lambda}, \boldsymbol{R})\,\underline{\mathbf{S}}_{ijk} + \sum_{a=1}^{8} O^{\mathsf{A}\,a}(\boldsymbol{\rho}, \boldsymbol{\lambda}, \boldsymbol{R})\,\underline{\mathbf{O}}^{\mathsf{A}\,a}_{ijk}$$

$$+ \sum_{a=1}^{8} O^{\mathsf{S}\,a}(\boldsymbol{\rho}, \boldsymbol{\lambda}, \boldsymbol{R})\,\underline{\mathbf{O}}^{\mathsf{S}\,a}_{ijk} + \sum_{\delta=1}^{10} \Delta^{\delta}(\boldsymbol{\rho}, \boldsymbol{\lambda}, \boldsymbol{R})\,\underline{\boldsymbol{\Delta}}^{\delta}_{ijk} \quad i, j = 1, 2, 3$$

pNRQCD ($mv \gg \Lambda_{\rm QCD}$)

$$\underline{\mathbf{S}}_{ijk} = \frac{1}{\sqrt{6}}\epsilon_{ijk}\,,$$

$$\underline{\mathbf{O}}^{\mathsf{A}\,a}_{ijk} = \frac{1}{2}\sum_{n=1}^{3}\epsilon_{ijn}\lambda^a_{kn}\,, \quad \underline{\mathbf{O}}^{\mathsf{S}\,a}_{ijk} = \frac{1}{2\sqrt{3}}\sum_{n=1}^{3}\left(\epsilon_{jkn}\lambda^a_{in} + \epsilon_{ikn}\lambda^a_{jn}\right)\,,$$

$$\underline{\mathbf{\Delta}}^1_{111} = \underline{\mathbf{\Delta}}^4_{222} = \underline{\mathbf{\Delta}}^{10}_{333} = 1\,,$$

$$\underline{\mathbf{\Delta}}^2_{112} = \underline{\mathbf{\Delta}}^2_{121} = \underline{\mathbf{\Delta}}^2_{211} = \underline{\mathbf{\Delta}}^3_{122} = \underline{\mathbf{\Delta}}^3_{212} = \underline{\mathbf{\Delta}}^3_{221} = \frac{1}{\sqrt{3}}\,,$$

$$\underline{\mathbf{\Delta}}^5_{113} = \underline{\mathbf{\Delta}}^5_{131} = \underline{\mathbf{\Delta}}^5_{311} = \underline{\mathbf{\Delta}}^7_{223} = \underline{\mathbf{\Delta}}^7_{232} = \underline{\mathbf{\Delta}}^7_{322} = \frac{1}{\sqrt{3}}\,,$$

$$\underline{\mathbf{\Delta}}^8_{133} = \underline{\mathbf{\Delta}}^8_{313} = \underline{\mathbf{\Delta}}^8_{331} = \underline{\mathbf{\Delta}}^9_{233} = \underline{\mathbf{\Delta}}^9_{323} = \underline{\mathbf{\Delta}}^9_{332} = \frac{1}{\sqrt{3}}\,,$$

$$\underline{\mathbf{\Delta}}^6_{123} = \underline{\mathbf{\Delta}}^6_{132} = \underline{\mathbf{\Delta}}^6_{213} = \underline{\mathbf{\Delta}}^6_{231} = \underline{\mathbf{\Delta}}^6_{312} = \underline{\mathbf{\Delta}}^6_{321} = \frac{1}{\sqrt{6}}\,,$$

all other entries are zero.

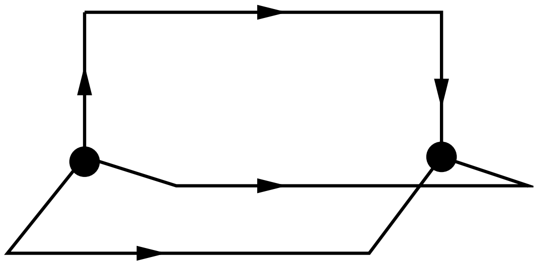
pNRQCD ($mv \gg \Lambda_{\rm QCD}$)

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \sum_{f=1}^{3} \bar{q}_f \, i \not{D} q_f$$

$$+\delta\mathcal{L}^{(0,0)}_{\text{pNRQCD}} + \delta\mathcal{L}^{(0,1)}_{\text{pNRQCD}} + \delta\mathcal{L}^{(1,0)}_{\text{pNRQCD}} + \cdots$$

pNRQCD ($mv \gg \Lambda_{\rm QCD}$)

$$\delta\mathcal{L}^{(0,0)}_{\textsf{pNRQCD}} = \int d^3\rho\, d^3\lambda \;\; S^\dagger \left[i\partial_0 - V_S^{(0)}\right] S + O^{\textsf{A}\dagger}\left[iD_0 - V_{O^{\textsf{A}}}^{(0)}\right] O^{\textsf{A}}$$
$$+O^{\textsf{S}\dagger}\left[iD_0 - V_{O^{\textsf{S}}}^{(0)}\right] O^{\textsf{S}} + \Delta^\dagger\left[iD_0 - V_{\Delta}^{(0)}\right]\Delta$$



$$V_S^{(0)}(\boldsymbol{\rho},\boldsymbol{\lambda}) = -\frac{2}{3}\,\alpha_{\rm s}\left(\frac{1}{|\boldsymbol{\rho}|} + \frac{1}{|\boldsymbol{\lambda}+\boldsymbol{\rho}/2|} + \frac{1}{|\boldsymbol{\lambda}-\boldsymbol{\rho}/2|}\right)$$

$$V_{O^{\textsf{A}}}^{(0)}(\boldsymbol{\rho},\boldsymbol{\lambda}) = -\frac{2}{3}\alpha_{\rm s}\left(\frac{1}{|\boldsymbol{\rho}|} - \frac{1}{8}\frac{1}{|\boldsymbol{\lambda}+\boldsymbol{\rho}/2|} - \frac{1}{8}\frac{1}{|\boldsymbol{\lambda}-\boldsymbol{\rho}/2|}\right)$$

$$V_{O^{\textsf{S}}}^{(0)}(\boldsymbol{\rho},\boldsymbol{\lambda}) = \frac{\alpha_{\rm s}}{3}\left(\frac{1}{|\boldsymbol{\rho}|} - \frac{5}{4}\frac{1}{|\boldsymbol{\lambda}+\boldsymbol{\rho}/2|} - \frac{5}{4}\frac{1}{|\boldsymbol{\lambda}-\boldsymbol{\rho}/2|}\right)$$

$$V_{\Delta}^{(0)}(\boldsymbol{\rho},\boldsymbol{\lambda}) = \frac{\alpha_{\rm s}}{3}\left(\frac{1}{|\boldsymbol{\rho}|} + \frac{1}{|\boldsymbol{\lambda}+\boldsymbol{\rho}/2|} + \frac{1}{|\boldsymbol{\lambda}-\boldsymbol{\rho}/2|}\right)$$

pNRQCD ($mv \gg \Lambda_{\rm QCD}$)

$$\delta\mathcal{L}^{(0,1)}_{\textsf{pNRQCD}} = \int d^3\rho\, d^3\lambda\, V^{(0,1)}_{S\boldsymbol{\rho}\cdot\boldsymbol{E}O^{\textsf{S}}}\frac{1}{2\sqrt{2}}\left[S^\dagger\boldsymbol{\rho}\cdot g\boldsymbol{E}^a\, O^{\textsf{S}\,a} + O^{\textsf{S}\,a\dagger}\boldsymbol{\rho}\cdot g\boldsymbol{E}^a\, S\right]$$

$$-V^{(0,1)}_{O^{\textsf{A}}\boldsymbol{\rho}\cdot\boldsymbol{E}O^{\textsf{S}}}\left(\frac{if^{abc}+3d^{abc}}{4\sqrt{3}}\right)\left[O^{\textsf{A}\,a\dagger}\boldsymbol{\rho}\cdot g\boldsymbol{E}^b\, O^{\textsf{S}\,c} + O^{\textsf{S}\,a\dagger}\boldsymbol{\rho}\cdot g\boldsymbol{E}^b\, O^{\textsf{A}\,c}\right]$$

$$+V^{(0,1)}_{O^{\textsf{A}}\boldsymbol{\rho}\cdot\boldsymbol{E}\Delta}\left[\left(\sum_{ii'jj'k=1}^{3}\epsilon_{ijk}T^a_{ii'}T^b_{jj'}\underline{\boldsymbol{\Delta}}^\delta_{i'j'k}\right)O^{\textsf{A}\,a\dagger}\boldsymbol{\rho}\cdot g\boldsymbol{E}^b\,\Delta^\delta\right.$$

$$\left.-\left(\sum_{ii'jj'k=1}^{3}\underline{\boldsymbol{\Delta}}^\delta_{ijk}T^b_{ii'}T^a_{jj'}\epsilon_{i'j'k}\right)\Delta^{\delta\dagger}\boldsymbol{\rho}\cdot g\boldsymbol{E}^b\, O^{\textsf{A}\,a}\right]$$

$$-V^{(0,1)}_{S\boldsymbol{\lambda}\cdot\boldsymbol{E}O^{\textsf{A}}}\frac{1}{\sqrt{6}}\left[S^\dagger\boldsymbol{\lambda}\cdot g\boldsymbol{E}^a\, O^{\textsf{A}\,a} + O^{\textsf{A}\,a\dagger}\boldsymbol{\lambda}\cdot g\boldsymbol{E}^a\, S\right]$$

$$-V^{(0,1)}_{O^{\textsf{A}}\boldsymbol{\lambda}\cdot\boldsymbol{E}O^{\textsf{A}}}\left(if^{abc}\frac{2m-m_3}{2m_R}+\frac{d^{abc}}{2}\right)O^{\textsf{A}\,a\dagger}\boldsymbol{\lambda}\cdot g\boldsymbol{E}^b\, O^{\textsf{A}\,c}+\cdots$$

pNRQCD ($mv \gg \Lambda_{\rm QCD}$)

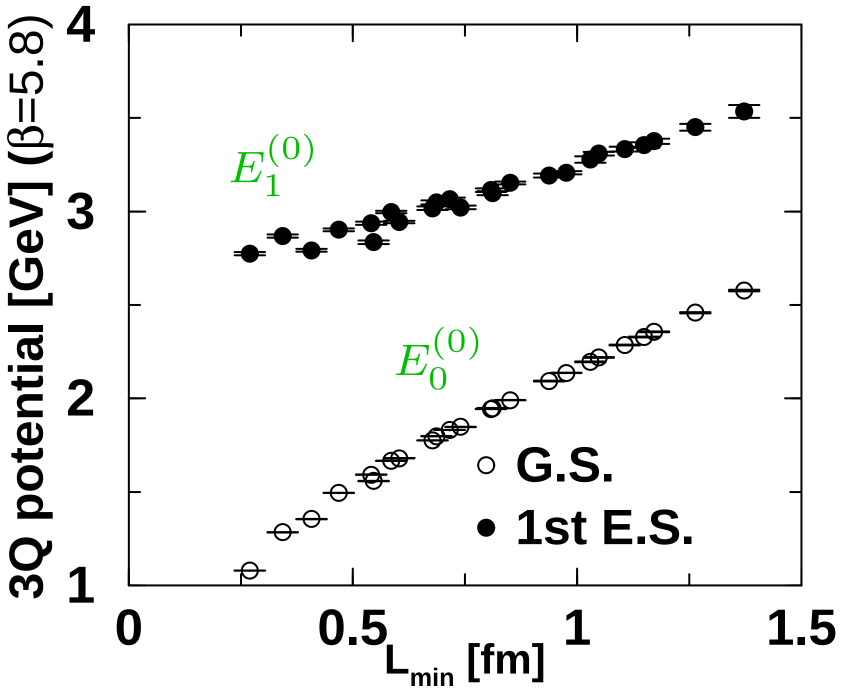
$$\begin{aligned}
\delta\mathcal{L}^{(1,0)}_{\rm pNRQCD} \quad = \quad & \int d^3\rho\, d^3\lambda \;\; S^\dagger \left[\frac{\boldsymbol{\nabla}_R^2}{2m_R} + \frac{\boldsymbol{\nabla}_\rho^2}{2m_\rho} + \frac{\boldsymbol{\nabla}_\lambda^2}{2m_\lambda} \right] S \\
& + O^{\mathsf{A}\dagger} \left[\frac{\boldsymbol{D}_R^2}{2m_R} + \frac{\boldsymbol{\nabla}_\rho^2}{2m_\rho} + \frac{\boldsymbol{\nabla}_\lambda^2}{2m_\lambda} \right] O^{\mathsf{A}} \\
& + O^{\mathsf{S}\dagger} \left[\frac{\boldsymbol{D}_R^2}{2m_R} + \frac{\boldsymbol{\nabla}_\rho^2}{2m_\rho} + \frac{\boldsymbol{\nabla}_\lambda^2}{2m_\lambda} \right] O^{\mathsf{S}} \\
& + \Delta^\dagger \left[\frac{\boldsymbol{D}_R^2}{2m_R} + \frac{\boldsymbol{\nabla}_\rho^2}{2m_\rho} + \frac{\boldsymbol{\nabla}_\lambda^2}{2m_\lambda} \right] \Delta + \cdots
\end{aligned}$$

pNRQCD ($mv \sim \Lambda_{\mathrm{QCD}}$)

- All scales above mv^2 are integrated out.

pNRQCD ($mv \sim \Lambda_{\rm QCD}$)

- All scales above mv^2 are integrated out.
- All gluonic excitations between heavy quarks are integrated out since they develop a gap of order $\Lambda_{\rm QCD}$ with the static QQQ energy.



Suganuma et al. 04

($r_0 \simeq 0.5$ fm)

pNRQCD ($mv \sim \Lambda_{\rm QCD}$)

- All scales above mv^2 are integrated out.
- All gluonic excitations between heavy quarks are integrated out since they develop a gap of order $\Lambda_{\rm QCD}$ with the static QQQ energy.

$\Rightarrow$ The singlet QQQ field S of energy mv^2 and momentum mv is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

pNRQCD ($mv \sim \Lambda_{\rm QCD}$)

$$\mathcal{L}_{\rm pNRQCD} = S^\dagger \left[i\partial_0 + \frac{\boldsymbol{\nabla}_R^2}{2m_R} + \frac{\boldsymbol{\nabla}_\rho^2}{2m_\rho} + \frac{\boldsymbol{\nabla}_\lambda^2}{2m_\lambda} - V^{(S)}(\rho, \lambda) \right] S$$

- The potential V_s is non-perturbative:

  (a) to be determined from the lattice;

  (b) to be determined from QCD vacuum models.

$$V^{(S)} = V_0^{(S)} + V_1^{(S)} + V_2^{(S)} + \cdots$$

$$V_0^{(S)} = \lim_{T\to\infty} \frac{i}{T} \ln \left\langle W_{QQQ}^{S} \right\rangle$$

lattice data in Takahashi Suganuma 04, Alexandrou et al. 02

$$V_1^{(\mathcal{S})} = -\sum_{i=1}^{3} \frac{1}{2m_i} \int_0^\infty dt\, t\, \langle\langle g\boldsymbol{E}(\boldsymbol{x}_i,t)\cdot g\boldsymbol{E}(\boldsymbol{x}_i,0)\rangle\rangle^{\mathcal{S}}_{c,QQQ}$$

$$\begin{aligned}
V^{(\mathcal{S})}_{\text{spin dep.}} \quad = \quad & \sum_{i=1}^{3} \frac{c_S^{(i)}}{4m_i^2} \boldsymbol{\sigma}^{(i)} \cdot \left[(\boldsymbol{\nabla}_{\boldsymbol{x}_i} V_S^{(0)}) \times (-i\boldsymbol{\nabla}_{\boldsymbol{x}_i}) \right] \\
& + \sum_{i,i'=1}^{3} i \frac{c_F^{(i)}}{m_i m_{i'}} \int_0^\infty dt\, t \sum_{kl=1}^{3} \langle\langle g\boldsymbol{B}^k(\boldsymbol{x}_i,t)\, g\boldsymbol{E}^l(\boldsymbol{x}_{i'},0)\rangle\rangle^{\mathcal{S}}_{c,QQQ} \boldsymbol{\sigma}_k^{(i)} (-i\boldsymbol{\nabla}^l_{\boldsymbol{x}_{i'}}) \\
& - \sum_{i>i'=1}^{3} i \frac{c_F^{(i)} c_F^{(i')}}{2m_i m_{i'}} \int_0^\infty dt \sum_{kl=1}^{3} \langle\langle g\boldsymbol{B}^k(\boldsymbol{x}_i,t)\, g\boldsymbol{B}^l(\boldsymbol{x}_{i'},0)\rangle\rangle^{\mathcal{S}}_{c,QQQ} \boldsymbol{\sigma}_k^{(i)} \boldsymbol{\sigma}_l^{(i')} \\
& - \sum_{i>i'=1}^{3} \left(d^{sv}_{Q_iQ_{i'}} + d^{vv}_{Q_iQ_{i'}} \langle\langle T^{a\,(i)} T^{a\,(i')} \rangle\rangle^{\mathcal{S}}_{c,QQQ} \right) \boldsymbol{\sigma}^{(i)} \cdot \boldsymbol{\sigma}^{(i')} \delta^3(\boldsymbol{x}_i - \boldsymbol{x}_{i'})
\end{aligned}$$

4. Outlook

Non Relativistic Effective Field Theories provide a systematic tool to study heavy baryons made of two or three heavy quarks.

The studies are just at the beginning and several developments are still possible.

In the case of QQq baryons; e.g.

- study of doubly charmed baryons in the completely non-perturbative case $mv \sim \Lambda_{\rm QCD}$. In particular, lattice studies of the potentials;
- construction of pNRQCD + light quark sector that fully implements chiral symmetry in order to addres pion transitions, decay and production processes.
- However, first of all, experimental confirmations are needed.

In the case of QQQ baryons only lattice studies are possible at the moment; e.g.

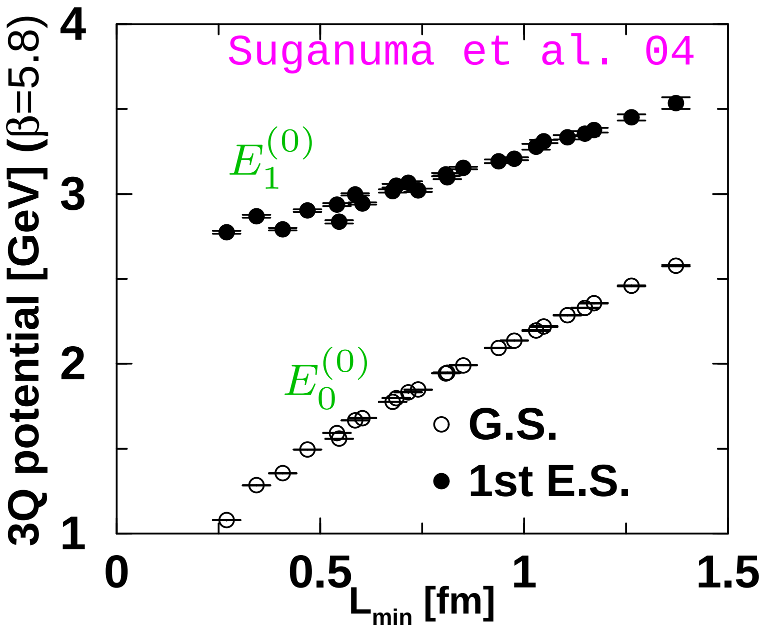
- determination on the lattice of the QQQ spin interaction;

- the study of the full spectrum of QQQ gluonic excitations.
  This will become soon available.
  Morningstar et al. 05(?)

The short-range spectrum of QQQ gluonic excitations is of the form:

$$E_\Lambda^{(S)} = V_0^{(S)}(\rho,\lambda) + \Lambda + \mathcal{O}(\lambda^2,\rho^2); \quad E_\Lambda^{(O^A)} = V_0^{(O^A)}(\rho,\lambda) + \Lambda + \mathcal{O}(\lambda^2,\rho^2)$$

$$E_\Lambda^{(O^S)} = V_0^{(O^S)}(\rho,\lambda) + \Lambda + \mathcal{O}(\lambda^2,\rho^2); \quad E_\Lambda^{(\Delta)} = V_0^{(\Delta)}(\rho,\lambda) + \Lambda + \mathcal{O}(\lambda^2,\rho^2)$$



(1) *Why does the energy just depend on one single coordinate L_{min}?*

(2) *What kind of excitation is $E_1^{(0)}$?*

Is it of the $E_\Lambda^{(S)}$, $E_\Lambda^{(O^A)}$, $E_\Lambda^{(O^S)}$ or $E_\Lambda^{(\Delta)}$ type?

Only more short-range data may solve the question.

(3) *One-loop determinations of $V^{(S)}$, $V^{(O^A)}$, $V^{(O^S)}$ and $V^{(\Delta)}$.*